# A Beckerian Theory of Taxation*

Margaret Chen
University of Chicago

Mikhail Golosov
University of Chicago

Ilia Krasikov
Arizona State University

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**Abstract**

We study taxation in matching environments in which both equilibrium sorting and the sharing of match surplus are endogenous. Tax reforms lead to complex adjustments in sorting and surplus sharing patterns across the whole economy. We develop a general approach for characterizing these equilibrium adjustments using matching elasticities, which are primitive objects that describe how agents' matching choices are affected by pecuniary payoffs in partial equilibrium. Applying this approach to a workhorse model of family formation, we first show that the sorting and surplus sharing responses are closely linked to complementarities in the creation of economic surplus, as first emphasized by Becker (1973). We then describe the effects of arbitrary tax reforms on tax revenues, the utilities of different agents, and social welfare; design tests to assess Pareto efficiency of existing tax systems; derive formulas that characterize optimal taxes; and propose numerical techniques to compute them. In a calibrated model, we find that optimal marginal tax rates are substantially lower and less progressive than in the Mirrleesian benchmark.

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# 1 Introduction

In his seminal work, Gary Becker (Becker (1973)) showed that match formation and within-match resource allocation are determined by the surplus that matches generate and by complementarities in producing that surplus. Although Becker originally focused on marriage markets, a large subsequent literature has extended his insights to many other settings, including the analysis of sorting and matching between firms and workers, workers and cities, and corporations and CEOs.

While endogenous sorting, surplus sharing, and complementarities are ubiquitous, the effects of taxation in such environments are not well understood. In this paper we develop a general approach to study the effects of taxation in a large class of endogenous matching models. We seek to understand what determines the costs and benefits of income taxation in such settings, how statutory tax burdens are shared among economic agents, whether existing taxes are efficient, and how the planner should design optimal taxes.

We develop our approach in a canonical model of family formation. Individuals who differ in labor market productivity form couples, choosing spouses based on both pecuniary utilities and non-pecuniary preference shocks. Pecuniary utilities are determined by consumption and labor supplies in the match. The labor supplies of the two spouses are potentially interdependent through home production, as in Becker (1973). Non-pecuniary preference shocks are drawn from a general distribution, which allows for preferences for marriage partners of particular types. The government imposes an arbitrary, potentially non-linear tax on the earnings of the two spouses. Equilibrium sorting, intra-family consumption sharing, and pecuniary utilities are all determined endogenously, as in standard frictionless matching models with transferable utility.

We first illustrate the key mechanisms in a simple two-type version of the model that admits a closed-form solution. We consider the effects of a tax reform in the spirit of Saez (2001), in which the government slightly raises the marginal tax rate in a small neighborhood of some earnings level. Mechanically, the reform increases taxes on all individuals with earnings above that earnings level. The individuals whose marginal taxes increase reduce their labor supply, thereby lowering tax revenues, to which we refer as the direct behavioral effect. In classical Mirrleesian settings, these two effects fully characterize the consequences of tax reforms.

This reform has three additional effects in matching environments. First, the spouse of the individual who reduces their earnings also adjusts their own labor supply. We refer to this response as the indirect behavioral effect. Second, the tax reform leads to the dissolution of some matches and the formation of others. Third, as marriage patterns change, consumption sharing adjusts throughout the economy, even within couples not directly affected by the reform. We refer to these last two responses as the sorting and sharing effects, respectively.

All three novel effects are closely linked to complementarities in labor supply, which is why we refer to them as Beckerian effects. Without complementarities, all three are absent and the tax analysis reduces to the classical Mirrleesian approach. Under positive complementarities, a higher labor supply of one spouse increases the other spouse's incentive to work, and earnings and tax revenues are higher when matching is more assortative. In this case, the indirect behavioral effect amplifies the distortionary costs of taxation. Moreover, taxing high earners leads to less assortative matching, thereby reducing aggregate earnings and tax revenues. Less assortative matching also results in more equal consumption sharing within couples, which the social planner who cares about inequality may find valuable. Under negative complementarities, a higher labor supply of one spouse reduces the other spouse's incentive to work, and earnings and tax revenues are higher when matching is less assortative. In this case, the indirect behavioral effect offsets some of the distortionary costs of taxation, but taxing high earners both reduces tax revenues and makes consumption sharing less equal within couples.

Motivated by this example, we develop a general approach for characterizing the effects of arbitrary tax reforms. The direct and indirect behavioral effects are captured, for each individual, by a pair of elasticities—one with respect to their own after-tax earnings and one with respect to the partner's after-tax earnings. These elasticities summarize local properties of preferences and are direct analogues of compensated elasticities in traditional tax analysis. The sorting and sharing effects are fundamentally different. A tax reform reshapes equilibrium matching patterns and within-family surplus division not only for couples directly affected by the reform but also for couples whose statutory taxes do not change. As a result, these effects cannot be studied using off-the-shelf techniques. A central methodological contribution of our paper is to develop a general approach for characterizing these two effects.

We show that the sorting and sharing effects are both determined by what we call matching elasticities, which measure how the probability that an individual matches with a given partner type changes when another potential partner offers them an additional dollar of consumption. Much like behavioral labor supply elasticities, which summarize local properties of preferences and surplus production, matching elasticities summarize local properties of the distribution of non-pecuniary shocks. Importantly, matching elasticities, just like behavioral elasticities, are partial equilibrium objects, which makes them straightforward to compute; for the shock distributions commonly used in applied work, they admit closed-form expressions.

Using matching elasticities, we describe how agents respond to changes in family surplus by deciding how to share them, whether to leave their current match, and whom to marry if they do. We show that the equilibrium sharing response can be found by solving an integral equation whose coefficients are constructed solely from matching elasticities. Once that solution is found, a separate integral equation, also built from matching elasticities, determines the equilibrium sorting response to any tax reform.

With this characterization, we describe the economic incidence of arbitrary tax reforms and their effects on tax revenues and welfare. We illustrate this approach by studying the two systems of taxing married individuals most commonly used in practice: individual-based taxation, in which the earnings of each spouse are taxed independently, and family-based taxation, in which taxes are imposed on the sum of spousal earnings. In both cases, we derive optimal tax formulas for taxes that maximize a given government objective and design efficiency tests that detect whether an existing tax schedule admits Pareto-improving reforms. We also show how the linear operators developed in the theoretical analysis can be used to compute optimal taxes numerically.

The techniques we develop extend with minimal modifications to many other environments with endogenous matching and consumption sharing, including models with rich heterogeneity, imperfectly transferable utility, competitive and non-competitive surplus sharing, and one-to-many matching. We illustrate such extensions both in a family economics setting, where matching occurs between heterogeneous spouses, and in a labor economics setting, where matching occurs between heterogeneous firms and workers.

To illustrate the quantitative implications of our approach, we study optimal individual-based taxation of married individuals. We use the findings of Calvo et al. (2024), who estimate complementarities in home production using German longitudinal panel data on married couples, to discipline the strength of labor supply complementarities in our model. We find that optimal tax rates are substantially lower and less progressive than in an otherwise identical economy that abstracts from complementarities. This result reflects several forces. Positive complementarities in home production make taxation more distortionary through the indirect behavioral effect. The sorting and sharing effects work in opposite directions across much of the earnings distribution, but for high earners, the losses from the sorting effect outweigh the gains from the sharing effect.

Our paper is motivated by a large empirical literature that has documented assortative matching patterns in a wide variety of settings. For example, higher educated and higher-earning individuals tend to marry higher educated and higher-earning spouses (Choo and Siow (2006), Fernández et al. (2005)); more productive employees tend to match with more productive firms (Bonhomme et al. (2019), Lamadon et al. (2022)); and more skilled workers are more likely to reside in cities with a higher share of highly educated residents (Diamond (2016), Diamond and Gaubert (2022)). Our baseline model builds on the canonical framework of marriage markets extensively studied in the family economics literature (Choo and Siow (2006), Chiappori et al. (2017), Galichon and Salanié (2022)), into which we introduce labor income taxation. Several intermediate steps of our analysis—such as the representation of the model with a continuum of types and the characterization of spousal choice probabilities in terms of elasticities—draw on the insights of Dupuy and Galichon (2014).

The closest papers to ours are Sachs et al. (2020), Scheuer and Werning (2017), Ales and Sleet (2016), and Ales and Sleet (2022). Like us, Ales and Sleet (2016) and Scheuer and Werning (2017) are motivated by taxation in Beckerian environments; however, both impose assumptions that guarantee perfectly assortative matching (PAM) in equilibrium. Under these assumptions, taxes do not affect sorting and operate only through within-match surplus division. Ales and Sleet (2022) examine optimal taxation in discrete-choice environments with Gumbel-type shocks. These models feature choices shaped by both observed characteristics and idiosyncratic non-pecuniary preferences—analogous to marriage decisions driven by both pecuniary and non-pecuniary factors. Our framework, however, introduces several additional layers that naturally arise in Beckerian settings: two- and multi-sided matching, rich non-discrete productivity heterogeneity, intensive margin labor-supply choices, arbitrary distributions of non-pecuniary shocks, and production complementarities. As a result, both the analytical tools and the economic mechanisms differ substantially. Sachs et al. (2020) study taxation in the presence of general-equilibrium effects and show that characterizing those effects requires solving for a fixed point of a linear system. In our setting, linear operators that characterize sorting and sharing responses also arise, but the structure of the underlying equations is different.

The normative literature on taxation in Beckerian environments without PAM is relatively scarce. Gayle and Shephard (2019) numerically solve for optimal taxes in a model of family formation and home production, but they do not have an analytical characterization and restrict both the matching process and the tax schedule to low-dimensional objects. Dupuy et al. (2020) provide analytical results, but only in an economy with two discrete types. Lamadon et al. (2022) numerically examine linear taxes in a worker–firm matching model with monopsonistic firms, and Berger et al. (2026) quantitatively study optimal tax progressivity, within the parametric class of Heathcote et al. (2017), when firms are oligopsonistic.

The paper is organized as follows. In Section 2, we present our baseline environment. In Section 3, we use a simple two-type economy to illustrate heuristically the key mechanisms through which complementarities shape the effects of taxation. In Section 4, we develop a general approach for characterizing how tax reforms affect equilibrium sorting and consumption sharing. In Section 5, we apply these techniques to study individual- and family-based taxation, derive optimal tax formulas, and design tests for Pareto efficiency of existing taxes. We illustrate the quantitative implications in Section 6. In Section 7, we show how to extend our techniques to other settings, including models with imperfectly transferable utility, arbitrary dimensions of heterogeneity, and firm–worker matching environments.

# 2 The baseline environment

The goal of our paper is to develop a general approach to studying taxation in a broad class of economies with endogenous matching and surplus sharing. We start by presenting key ideas in a simple model of couple formation and home production. We refer to this model as the *baseline environment*.

In our baseline environment, the economy is populated by a unit mass of heterogeneous individuals who form couples. Events unfold in two stages. In the first stage, each individual chooses a marriage partner. When choosing a partner, an individual considers both pecuniary utilities, i.e., utilities from consumption and labor that they can obtain from marrying partners of different types, and idiosyncratic non-pecuniary preference shocks. In the second stage, married couples produce family surplus. They pay taxes on their market earnings, purchase consumption goods with after-tax income, and allocate those goods between spouses. The allocation of consumption and labor determines pecuniary utilities. In equilibrium, these pecuniary utilities coincide with agents' expectations in the first stage.

We use $w$ to denote an individual's type. The distribution of types among agents is denoted by $G$. We assume that this distribution satisfies one of two sets of assumptions, to which we refer as the *continuum-* and *finite-type* cases. In the continuum-type case, $G$ is supported on a compact interval in $\mathbb{R}_{++}$ and admits a density. In the finite-type case, $G$ assigns all mass to a finite number of points in $\mathbb{R}_{++}$. Regardless of whether we consider the continuum- or finite-type case, we use the same notation $g \equiv (g(w))_w$ to describe the distribution of types, with the understanding that $g$ is a continuous probability density function in the former case and a probability mass function in the latter case. In either case, $g$ is strictly positive on the support of $G$.

## 2.1 The marriage decision problem

In this section, we describe the spousal choice problem. Let $z \equiv (z(w'))_{w'}$ be pecuniary utilities that an individual can obtain from marrying spouses of different types and $\epsilon \equiv (\epsilon(w'))_{w'}$ be their non-pecuniary preference shocks for spouses of different types.[1] The marriage partner choice of this individual is described by the following maximization problem:

$$\max_{w'} z(w') + \beta^{-1}\epsilon(w'). \tag{1}$$

In this problem, the parameter $\beta \in (0, \infty)$ captures the relative importance of pecuniary motives in finding a marriage partner: individuals choose partners solely for non-pecuniary reasons as $\beta \to 0$, and for pecuniary reasons as $\beta \to \infty$.

[1] In the finite-type case, $z$ is a vector with one coordinate per type and $\epsilon$ is a random vector of the same dimension. In the continuum-type case, $z$ is a continuous function on the support of $G$ and $\epsilon$ is a random field indexed by $w'$. See the appendix for additional details.

All individuals of the same type face the same menu of pecuniary utilities, but realizations of the non-pecuniary preference shocks $\epsilon$ are idiosyncratic. We use $\Psi_w$ to denote the probability law from which type $w$ draws $\epsilon$. We impose little structure on this law apart from mild technical conditions stated in the appendix. Let

$$U_w(z) := \mathbb{E}_w \left[ \max_{w'} z(w') + \beta^{-1} \epsilon(w') \right] \tag{2}$$

denote the ex-ante expected utility of type $w$ who faces a menu of pecuniary utilities $z$ before non-pecuniary shocks are realized. The function $U_w$ is determined by $\Psi_w$ and is, therefore, a primitive object in our environment.[2]

In applied work, researchers usually choose specifications for the law $\Psi_w$ under which $U_w$ has an explicit analytical expression. For example, the canonical logit model, first introduced by McFadden (1974) and extensively used in the family economics literature since at least Choo and Siow (2006), assumes that preference shocks are drawn independently for each $w'$ from the EV-I distribution with type-specific location parameters.[3] In the finite-type case, $U_w$ then takes the form

$$U_w(z) = \beta^{-1} \ln \left( \textstyle\sum_{w'} \exp(\beta z(w')) g(w') \right) + \text{const.} \tag{3}$$

Its continuum-type version is

$$U_w(z) = \beta^{-1} \ln \left( \textstyle\int \exp(\beta z(w')) g(w') dw' \right) + \text{const},$$

which is the same expression as (3) but with an integral replacing the sum. Davis and Schiraldi (2014), Dupuy and Galichon (2014), and Galichon and Salanié (2022) provide explicit expressions for a variety of other commonly used distributions of non-pecuniary shocks.

We use $\partial_{w'} U_w(z)$ to denote the partial derivative of $U_w(z)$ with respect to $z(w')$. By the envelope theorem, $\partial_{w'} U_w(z)$ is equal to the ex-ante probability that a person of type $w$ chooses a partner of type $w'$.[4] For commonly used laws of preference shocks, this partial derivative can be written explicitly. For example, in the logit model it is given by

$$\partial_{w'} U_w(z) = \frac{\exp(\beta z(w')) g(w')}{\sum_{\tilde{w}'} \exp(\beta z(\tilde{w}')) g(\tilde{w}')} \quad \text{or} \quad \frac{\exp(\beta z(w')) g(w')}{\int \exp(\beta z(\tilde{w}')) g(\tilde{w}') d\tilde{w}'}, \tag{4}$$

depending on whether one uses the finite- or continuum-type specification.

[2] In our formulation, we do not normalize the shocks $\epsilon$ to have mean zero. Equivalently, we can write (2) as $U_w(z) = \mathbb{E}_w \left[ \max_{w'} z(w') + \Phi_w(w') + \beta^{-1} \tilde{\epsilon}(w') \right]$, where $\mathbb{E}_w \tilde{\epsilon}(w') = 0$ for all $w'$ and $\Phi_w(w')$ is an exogenous function that captures non-pecuniary benefits of matches, such as non-pecuniary preferences for partners of particular types.

[3] The location parameter of the shock for type $w'$ is $\ln g(w')$, which produces the weights $g(w')$ in (3) and its continuum-type counterpart. This centering ensures that, when pecuniary utilities are the same in all matches, an individual is equally likely to match with any person in the economy; see Dagsvik (1994, 2000).

[4] In the continuum-type case, $U_w$ is a functional on the space of continuous functions equipped with the sup norm, and $(\partial_{w'} U_w)_{w'}$ denotes its Fréchet derivative, which takes the form of a probability density function under our regularity conditions; see the appendix for details.

## 2.2 Pecuniary utilities and complementarities

In our baseline economy, we assume that the allocation of resources within families is efficient. In addition, we make several assumptions that simplify the exposition but are inessential for our results (see Section 7 for various extensions of the baseline economy). First, we assume that preferences of individuals are quasi-linear in consumption. Second, we assume that individuals are heterogeneous only in labor market productivity, so that a person of type $w$ who supplies $l$ hours of labor earns $y = wl$.

Under quasi-linearity, all efficient allocations of labor supplies $(l, l')$ and consumption $(c, c')$ of the two spouses solve the problem of maximizing the sum of spousal utilities. We denote taxes on the earnings of the two spouses by $\mathcal{T}(y, y')$ and write the couple's maximization problem as

$$v(w, w') := \max_{(c,c',l,l')} c + c' + \mathcal{N}(l, l') \tag{5}$$

subject to the budget constraint

$$c + c' \leq wl + w'l' - \mathcal{T}(wl, w'l'), \tag{6}$$

where $\mathcal{N}$ is a twice continuously differentiable, strictly concave function that captures the joint (dis)utility of labor of the two spouses. The maximization problem (5) pins down the optimal labor supplies $(l, l')$ of the two spouses and their total consumption $c + c'$. All efficient intra-family allocations are characterized by the same labor supplies and total consumption, and all feasible divisions of that consumption into individual consumption pairs trace the intra-family Pareto frontier. We refer to $v$ defined in (5) as family surplus.

We require both $\mathcal{N}$ and $\mathcal{T}$ to be symmetric, which implies that $v$ is symmetric as well. The key economic content of this assumption is that the family tax depends only on the pair of earnings $(y, y')$ but not on which spouse earns which, nor on other characteristics, such as sex. We discuss other tax functions in environments with general multi-dimensional heterogeneity in Section 7.

The cross-partial derivative of $\mathcal{N}$ will play an important role in our tax analysis. We say that there are *no complementarities* if $\frac{\partial^2 \mathcal{N}}{\partial l \partial l'} = 0$, *positive complementarities* if $\frac{\partial^2 \mathcal{N}}{\partial l \partial l'} > 0$, and *negative complementarities* if $\frac{\partial^2 \mathcal{N}}{\partial l \partial l'} < 0$, for all $(l, l')$. Complementarities may arise through several channels, as we illustrate in the following examples.

**Example 1.** Consider a model of home production in the spirit of Becker (1973). Suppose all individuals derive utility from a market good $c$ and a home good $C$, and disutility from labor supply in the market $l$ and labor supply at home $m$. Individual preferences are given by $c + C - N(l + m)$, where $N$ is an increasing and strictly convex function. The home good is produced with technology $D(m, m')$, where $m$ and $m'$ are the home labor supplies of the two spouses, and $D$ is increasing, symmetric, and concave. A fraction $\kappa$ of the home good consumption is public within the couple, and $1 - \kappa$ is private. All efficient

allocations in such an economy solve

$$\max_{(c,c',l,l',m,m')} c + c' + (\kappa+1)D(m,m') - N(l+m) - N(l'+m') \tag{7}$$

subject to (6). Let

$$\mathcal{N}(l,l') := \max_{(m,m')} (\kappa+1)D(m,m') - N(l+m) - N(l'+m')$$

be the value of the optimal allocation of home labor supply and home consumption conditional on market labor supplies $(l,l')$. It is immediate that problem (7) can be written as (5), and that the cross-partial of $\mathcal{N}$ has the same sign as that of $D$.

**Example 2.** Suppose that each individual has preferences $c + N(\ell,\ell')$ over their own consumption $c$ and leisure $\ell$, as well as the leisure of their spouse $\ell'$, where the function $N$ is strictly increasing in $\ell$ and weakly increasing in $\ell'$. Each person has a unit time endowment that can be allocated between work and leisure. Spouses jointly choose their consumption, labor supplies, and leisure to maximize the sum of their utilities subject to the budget constraint (6). This maximization problem can be written as (5) with[5]

$$\mathcal{N}(l,l') := N(1-l,1-l') + N(1-l',1-l).$$

In this specification, the sign of the cross-partial of $\mathcal{N}$ is the same as that of $N$. Thus, complementarities are positive if spouses enjoy leisure activities more when they do them together.

We assume that the tax function $\mathcal{T}$ is twice continuously differentiable, and sufficiently well-behaved so that the optimal labor supplies are interior, unique for each spouse, and satisfy the second-order conditions strictly.[6] We do not impose any additional structure beyond these fairly standard regularity restrictions, and our main results are derived for tax functions that depend on the earnings of the two spouses in an arbitrary way. However, two special classes of tax functions—family-based and individual-based—are particularly important because they are common in practice, and we investigate them more closely. Under family-based taxation, taxes are imposed on the total earnings of the household, so that $\mathcal{T}$ has the form $\mathcal{T}(y,y') = T^{\mathrm{F}}(y+y')$ for some function $T^{\mathrm{F}}$. Under individual-based taxation, taxes are imposed on individual earnings separately, so that $\mathcal{T}$ takes the form $\mathcal{T}(y,y') = T(y) + T(y')$, where $T$ is the tax function on earnings of an individual.

The division of total consumption between spouses is endogenous and determined in equilibrium, and it will be more convenient to work not with the consumption of each spouse but with their pecuniary utilities. Let $u_w(w')$ be the pecuniary utility that an individual of type $w$ obtains in a match with $w'$. Feasibility requires that

$$u_w(w') + u_{w'}(w) = v(w,w') \ \ \forall w,w'. \tag{8}$$

[5] We assume that the time constraint never binds.

[6] The second-order condition is verified when $\mathcal{T}$ is not too concave relative to $\mathcal{N}$.

There is a one-to-one relationship between the pecuniary utilities $(u_w(w'), u_{w'}(w))$ and the consumption pairs that individuals $w$ and $w'$ obtain in their marriage.

## 2.3 Matching equilibrium

A given tax function $\mathcal{T}$ determines the family surplus $v$ by (5). We describe the equilibrium for any $v$ in terms of two objects: the collection of pecuniary utilities $u \equiv (u_w(w'))_{w,w'}$ and the collection of choice probabilities $q \equiv (q_w(w'))_{w,w'}$, where $q_w(w')$ describes the probability with which type $w$ marries type $w'$.

The equilibrium should satisfy three conditions. First, the pair of pecuniary utilities $(u_w(w'), u_{w'}(w))$ must be feasible, i.e., satisfy (8). Second, the choice probabilities $(q_w(w'))_{w'}$ must describe optimal choices for each type $w$ given the menu of pecuniary utilities $(u_w(w'))_{w'}$ that they face in equilibrium. Given our discussion in Section 2.1, this optimality condition can be written as

$$q_w(w') = \partial_{w'} U_w(u_w) \ \ \forall w, w'. \tag{9}$$

Finally, the marriage market must clear. Since non-pecuniary shocks are idiosyncratic, $q_w(w')$ also describes the fraction of type-$w$ individuals who choose to marry type $w'$. Therefore, the mass of type-$w$ individuals who marry type $w'$ is $q_w(w')g(w)$. By the same argument, the mass of type-$w'$ individuals who marry type $w$ is $q_{w'}(w)g(w')$. Since these are the same matches counted from the two sides, the marriage market clearing condition reads as

$$q_w(w')g(w) = q_{w'}(w)g(w') \ \ \forall w, w'. \tag{10}$$

These equations allow us to define the marriage market equilibrium as follows.[7]

**Definition.** An equilibrium in the marriage market given the family surplus $v$ consists of functions $(u, q)$ such that

(A). marriage choice probabilities $q$ are optimal given $u$, i.e., equation (9) holds;

(B). the marriage market clears, i.e., equation (10) holds;

(C). pecuniary utilities are feasible, i.e., equation (8) holds.

Our notion of equilibrium is mathematically equivalent to the notion of matching stability (Shapley and Shubik (1971)): under transferable utility, no pair of individuals can form a match that makes both better off than in equilibrium, once non-pecuniary shocks are taken into account (Galichon and Salanié (2022)). The function $f \equiv (f(w, w'))_{w,w'}$ defined by $f(w, w') := q_w(w')g(w)$ describes the joint distribution of matches. By (10), $f$ is symmetric; it is a probability density function in the continuum-

[7] We show in the appendix that the equilibrium is unique.

type case and a probability mass function in the finite-type case. We use $F$ to denote the cumulative distribution function associated with $f$.[8]

To build intuition for how matching is determined in equilibrium, it is helpful to consider several special cases. First, let $\beta \to \infty$. In this case, matching is determined solely by pecuniary motives, as in Becker (1973). Matching is easy to characterize when family surplus is either supermodular or submodular. Matching is perfectly positive assortative in the former case and perfectly negative assortative in the latter case; in both cases these equilibrium matching patterns maximize total surplus $\int v dF$. When $\beta$ is finite, sorting is imperfect. If there are no complementarities, taxes are individual-based, and non-pecuniary shocks are independent of productivities (i.e., the law $\Psi_w$ is the same for all $w$), then matching is random and the pecuniary utility $u_w(w')$ is the same for all $w'$. When complementarities are present or the law of non-pecuniary shocks varies with types, capturing, for example, non-pecuniary preferences for spouses of particular types, equilibrium sorting is generally imperfect.

# 3 An illustrative example

Before presenting our general approach, it will be helpful to discuss heuristically how the economic effects of taxation in our endogenous matching model differ from the effects emphasized by the traditional Mirrleesian tax literature.

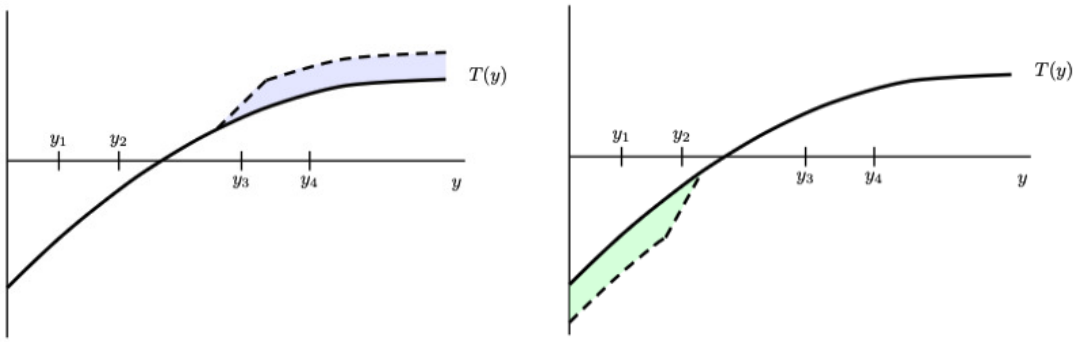
Suppose that agents can be of one of two types, $w \in \{L, H\}$, where $L < H$. In the two-type economy there are three kinds of couples: homogeneous couples $LL$ and $HH$, and heterogeneous couples $HL$. We assume that couples are subject to individual-based taxation $T$. There can be at most four distinct earnings levels, which we assume to be the case. We arrange them in increasing order as

$$y_1 < y_2 < y_3 < y_4.$$

Figure 1 plots a hypothetical tax schedule $T$ and these four earnings levels.

We illustrate the effects of taxation by considering a tax reform in the spirit of Saez (2001). Suppose that the government slightly increases the marginal tax rate on earnings in a small neighborhood of $y_3$, keeping all other marginal rates unchanged, as shown in Figure 1(a). In classical Mirrleesian settings, this reform has two effects. The *mechanical* effect captures the change in tax revenues in the absence of any equilibrium adjustments. In this example, it is equal to the increase in statutory taxes for $y_4$-earners. The *direct behavioral* effect captures how individuals change their labor supplies in response to changes in their taxes. In this example, it is determined by the reduction in the labor supply of

[8] We define the equilibrium in terms of pecuniary utilities $u$ and choice probability densities $q$, since understanding the properties of these objects is central to our characterization of optimal taxation. It is common in the literature (see, e.g., Chiappori and Salanié (2023)) to define a matching equilibrium in terms of $u$ and $f$. Since $f$ and $q$ are proportional to each other up to the exogenous $g$, the two ways of defining the equilibrium are equivalent.




(a) Increasing taxes on the rich, $y > y_3$. (b) Increasing transfers to the poor, $y < y_2$.

Figure 1: The tax reform in panel (a) increases taxes for individuals with earnings $y > y_3$ by slightly raising the marginal tax rate in a neighborhood of $y_3$. The tax reform in panel (b) increases transfers to individuals with earnings $y < y_2$. The phasing out of these transfers raises the effective marginal tax rate at $y_2$.

$y_3$-earners in response to the increase in their marginal tax rate. These two effects fully summarize the economic consequences of any tax reform in Mirrleesian settings.

Three additional, Beckerian, effects emerge in our economy. The reduction in the labor supply of $y_3$-earners affects the incentives of their spouses to supply labor even though the spouses are not directly affected by the tax reform. We refer to this earnings response as the *indirect behavioral* effect. Moreover, because of the tax increase, some $y_4$-earners may leave their matches and marry other types. This leads to changes in both equilibrium sorting and surplus sharing, to which we refer as the *sorting* and *sharing* effects.

To build intuition for the Beckerian effects and the role of complementarities, suppose that the tax function is affine with slope $\tau \in (0, 1)$ and that $\mathcal{N}$ is given by

$$\mathcal{N}(l, l') = -\tfrac{1}{2}(l)^2 - \tfrac{1}{2}(l')^2 + \phi l l',$$

where the parameter $\phi \in (-L/H, 1)$ captures complementarities in labor supply. Under this specification, the labor supply optimality condition for $w$ married to $w'$ is

$$l_w(w') = (1 - \tau)w + \phi l_{w'}(w). \tag{11}$$

This equation shows that a higher labor supply of the partner increases $w$'s incentives to supply labor under positive complementarities and decreases them under negative complementarities.

Combining this optimality condition with the optimality condition for $l_{w'}(w)$, we can solve for the optimal earnings as

$$y_w(w') = \frac{(1 - \tau)w}{1 - \phi^2}(w + \phi w'). \tag{12}$$

Equation (12) shows that if complementarities are positive, then earnings are ordered as

$$y_L(L) < y_L(H) < y_H(L) < y_H(H),$$

so that individuals in homogeneous matches are in the tails of the earnings distribution, and that the earnings function $y \equiv (y_w(w'))_{w,w'}$ is supermodular in $(w, w')$. Supermodularity implies that total earnings in the economy, and hence tax revenues, are higher if matching is more assortative. Under negative complementarities, earnings are ordered as

$$y_L(H) < y_L(L) < y_H(H) < y_H(L),$$

the earnings function is submodular, and earnings and tax revenues are higher when matching is less assortative.

These observations help to understand the Beckerian effects qualitatively. Suppose that complementarities are positive. Then the tax reform in Figure 1(a) increases the marginal tax rate at $y_3 = y_H(L)$. As these earners decrease their labor supply, their partners' earnings $y_2 = y_L(H)$ fall as well due to the indirect behavioral effect. Furthermore, $H$ types married to $H$ face higher total taxes after the reform, which increases the incentives for $H$ types to marry $L$. This raises $H$'s demand for $L$ types in the marriage market and increases the share of surplus that $L$ types obtain in $HL$ matches. Consequently, the tax reform makes matching less assortative, further reducing earnings and tax revenues through the indirect behavioral and sorting effects, but it increases intra-family redistribution from $H$ to $L$ types in $HL$ matches. The analysis of negative complementarities is similar. The indirect behavioral effect now raises tax revenues, as the reduction in the labor supply of each $y_3$-earner raises the labor supply of their spouse. The tax reform also reduces the incentives to form $HL$ matches, and it lowers the share of surplus that $L$ types get in such couples.

The sign of the indirect behavioral effect is always the same, irrespective of which agents face an increase in marginal taxes: it is negative (i.e., it reduces tax revenues) under positive complementarities and positive under negative complementarities. The same is true for the sharing effect, but the sign of the sorting effect depends on which agents face a tax increase. Suppose, for example, that the tax reform increases taxes for all earnings above $y_2$ or, analogously, increases transfers for all earnings below $y_2$, as depicted in Figure 1(b). Under positive complementarities, this reform makes forming $LL$ matches more attractive. Consequently, it makes matching more assortative, which raises earnings and tax revenues, and it increases $L$'s share of surplus in $HL$ matches. Under negative complementarities, the reform in Figure 1(b) makes forming $HL$ matches more attractive. Consequently, it makes matching less assortative, which again raises earnings and tax revenues, but it lowers the share of surplus that $L$ obtain in $HL$ matches, as $L$ types keep only a fraction of the transfer and their $H$ spouses get the rest. Thus, the sorting effects of the two reforms in Figure 1 have opposite signs, while their sharing effects have the same sign.

# 4 Tax reforms in matching environments

In the previous section, we discussed heuristically the effects of tax reforms in a two-type economy. We now consider tax reforms systematically in general settings. For concreteness, we focus on the analysis of the economy with a continuum of types; the analysis of the finite-type economy is very similar.

Let $\mathcal{T}$ be a tax function, $v$ the corresponding family surplus, $(u, q)$ the equilibrium, $f$ the joint distribution of matches, and $y \equiv (y_w(w'))_{w,w'}$ the optimal choices of earnings. Suppose that the government changes taxes to $\mathcal{T} + \varepsilon\dot{\mathcal{T}}$, where $\dot{\mathcal{T}}$ is an arbitrary symmetric,[9] twice continuously differentiable function and $\varepsilon$ is a scalar. All equilibrium objects are then functions of $\varepsilon$, and we use dots to denote their derivatives with respect to $\varepsilon$ at $\varepsilon = 0$; for example, $\dot{v}$ is the response of family surplus to the reform. As we explain in the appendix, these derivatives are well defined, since the equilibrium is differentiable in $\varepsilon$.

It will be helpful to introduce several conventions. For any variable that is a function of $(w, w')$, we introduce a swap operator, denoted by the superscript $\mathfrak{s}$, that swaps the indices $w$ and $w'$. For example, if $y_w(w')$ denotes the earnings of person $w$ married to $w'$, then $y_w^{\mathfrak{s}}(w') = y_{w'}(w)$ denotes earnings of the spouse of that person, i.e., of person $w'$ married to $w$. This convention allows us to write succinctly the earnings in all matches as $(y, y^{\mathfrak{s}})$. Symmetric functions are obviously unaffected by the swap operation, e.g., $v^{\mathfrak{s}} = v$.

Slightly abusing notation, we use the shorthand $\mathcal{T}(w, w')$ to denote $\mathcal{T}(y_w(w'), y_w^{\mathfrak{s}}(w'))$, i.e., the total pre-reform tax burden of family $(w, w')$. Similarly, we use $\dot{\mathcal{T}}(w, w')$ to denote $\dot{\mathcal{T}}(y_w(w'), y_w^{\mathfrak{s}}(w'))$. Note that $\dot{\mathcal{T}}(w, w')$ captures the statutory changes in taxes for couple $(w, w')$, i.e., the change in the couple's taxes evaluated at the pre-reform earnings. We apply the same convention to other functions of earnings, such as marginal tax rates, throughout the paper.

The effect of a tax reform on family surplus is easy to characterize using the envelope theorem, which yields

$$\dot{v} = -\dot{\mathcal{T}}, \tag{13}$$

in the sense that $\dot{v}(w, w') = -\dot{\mathcal{T}}(w, w')$ for all $(w, w')$. We now show how to characterize the behavioral, sorting and sharing effects of this tax reform.

## 4.1 Sorting and sharing effects

The choice probability $q_w(w')$ depends both on the pecuniary utility $u_w(w')$ offered in that match and on pecuniary utilities $(u_w(\tilde{w}'))_{\tilde{w}' \neq w'}$ available in alternative matches. Let $\gamma_w(\tilde{w}'|w')$ denote the

[9] Recall that symmetry captures the fact that the labeling of spouses is arbitrary.

semi-elasticity of $q_w(w')$ with respect to the pecuniary utility offered by an alternative match $\tilde{w}'$:

$$\gamma_w(\tilde{w}'|w') = -\partial_{\tilde{w}'} \ln q_w(w') := \frac{-\partial^2_{\tilde{w}'w'} U_w(u_w)}{\partial_{w'} U_w(u_w)} \text{ for all } \tilde{w}' \neq w'. \tag{14}$$

We refer to $\gamma \equiv (\gamma_w(\tilde{w}'|w'))_{w,w',\tilde{w}'\neq w'}$ as *matching elasticities.*[10] As we shall see, they are the key objects that determine sorting and sharing responses.

Recall that the function $U_w$ is fully determined by the law of non-pecuniary shocks $\Psi_w$. Thus, matching elasticities capture local properties of this law, summarized by the ratio $\frac{-\partial^2_{\tilde{w}'w'} U_w}{\partial_{w'} U_w}$. In this sense, they are analogous to price elasticities in consumer theory, which summarize consumer preferences in terms of certain derivatives of the utility function. Like price elasticities, matching elasticities in general depend on the point at which this ratio is evaluated. We evaluate those elasticities at the equilibrium corresponding to $\mathcal{T}$. The parallel with price elasticities extends further. Like price elasticities, matching elasticities are partial equilibrium objects: they summarize how an individual $w$ responds to person $\tilde{w}'$ offering an additional dollar of consumption, and do not consider why person $\tilde{w}'$ is offering that dollar or the equilibrium repercussions of $w$ taking that offer and forming a match with $\tilde{w}'$. One can in principle estimate matching elasticities directly, say, by instrumenting for changes in surplus sharing. For all commonly used multinomial logit models, matching elasticities are available in explicit form. For example, in the standard logit model, the matching elasticity is $\gamma_w(\tilde{w}'|w') = \beta q_w(\tilde{w}')$, which can be seen by differentiating (4).

Let

$$\bar{\gamma}_w(w') := \int \gamma_w(\tilde{w}'|w') d\tilde{w}',$$

which captures how the probability of $w$ matching with $w'$ changes if all types $\tilde{w}' \neq w'$ offer an extra dollar to $w$.[11] It is easy to see from equation (1) that an extra dollar offered by all potential spouses, including $\tilde{w}' = w'$, does not affect the optimal spousal choice. This implies that $\bar{\gamma}_w(w')$ also satisfies $\bar{\gamma}_w(w') = \partial_{w'} \ln q_w(w')$, i.e., it is the semi-elasticity of $q_w(w')$ with respect to $u_w(w')$. Furthermore, $\gamma$ is positive, so that $\frac{\gamma_w(\cdot|w')}{\bar{\gamma}_w(w')}$ describes the probabilities with which person $w$ marries other types if they respond to losing a dollar in surplus in their current match by leaving $w'$.

To find the sharing and sorting responses $(\dot{u}, \dot{q})$, we differentiate equations (8)–(10) and rearrange terms. To write the resulting equations succinctly, let $\dot{\pi}$ be defined by

$$\dot{\pi}_w(w') := \int \frac{\gamma_w(\tilde{w}'|w')}{\bar{\gamma}_w(w')} \dot{u}_w(\tilde{w}') d\tilde{w}'. \tag{15}$$

It captures the change in the expected value of the outside option of person $w$ who was married to $w'$ before the tax reform. It consists of changes in pecuniary gains that person $w$ can get in other matches,

[10] In the continuum-type case, $\left(\partial^2_{\tilde{w}'w'} U_w(u_w)\right)_{w',\tilde{w}'\neq w'}$ is the off-diagonal part of the second Fréchet derivative of $U_w$, which we assume is represented by a continuous kernel, so that $\gamma_w$ is a continuous function. We provide additional details in the appendix.

[11] The finite-type version of this definition is $\bar{\gamma}_w(w') := \sum_{\tilde{w}'\neq w'} \gamma_w(\tilde{w}'|w')$. The same comment applies to finite-type versions of all integrals that appear in this section.

$(\dot{u}_w(\tilde{w}'))_{\tilde{w}'\neq w'}$, weighted by the probabilities with which person $w$ leaves their current match with $w'$ for each alternative, $\left(\frac{\gamma_w(\tilde{w}'|w')}{\bar{\gamma}_w(w')}\right)_{\tilde{w}'\neq w'}$. Using $\dot{\pi}$, we can write the responses $\dot{u}$ and $\dot{q}$ as

$$\dot{u}_w(w') = \dot{\pi}_w(w') + \underbrace{\frac{\bar{\gamma}^{\mathfrak{s}}_w(w')}{\bar{\gamma}_w(w') + \bar{\gamma}^{\mathfrak{s}}_w(w')}}_{\text{bargaining weight}} \times (\dot{v}(w,w') - \dot{\pi}_w(w') - \dot{\pi}^{\mathfrak{s}}_w(w')) \tag{16}$$

and

$$\frac{\dot{q}_w(w')}{q_w(w')} = \underbrace{\frac{\bar{\gamma}_w(w')\bar{\gamma}^{\mathfrak{s}}_w(w')}{\bar{\gamma}_w(w') + \bar{\gamma}^{\mathfrak{s}}_w(w')}}_{\text{match sensitivity}} \times (\dot{v}(w,w') - \dot{\pi}_w(w') - \dot{\pi}^{\mathfrak{s}}_w(w')). \tag{17}$$

Equation (16) shows that the change in pecuniary utility of any type $w$ married to $w'$, $\dot{u}_w(w')$, depends on two terms: the change in the outside option, $\dot{\pi}_w(w')$, and a share of $\dot{v}(w,w') - \dot{\pi}_w(w') - \dot{\pi}^{\mathfrak{s}}_w(w')$. The latter term captures the change in the net surplus of the match, i.e., the difference between surplus $v(w,w')$ that $w$ and $w'$ obtain by forming a match and the sum of their outside options (recall that the superscript $\mathfrak{s}$ means that $\dot{\pi}^{\mathfrak{s}}_w(w') = \dot{\pi}_{w'}(w)$ is the change in the outside option of type $w'$ married to $w$). Changes in the net surplus are borne by spouses in proportion to $\frac{\bar{\gamma}^{\mathfrak{s}}_w(w')}{\bar{\gamma}_w(w')+\bar{\gamma}^{\mathfrak{s}}_w(w')}$ for person $w$ and $\frac{\bar{\gamma}_w(w')}{\bar{\gamma}_w(w')+\bar{\gamma}^{\mathfrak{s}}_w(w')}$ for their spouse, which can be interpreted as endogenous bargaining weights. These weights are determined by matching elasticities, with the less elastic spouse getting a higher weight. The reason is market clearing. The more elastic spouse's partner choice responds more strongly to their share of the change in net surplus, so the same share moves more of their type in or out of the match than it moves their partner's type. To keep the two sides of the match in balance, the less elastic spouse must bear the larger share.

Equation (17) shows that sorting responses are also determined by changes in net surplus. The fraction of $(w,w')$ matches that dissolve in response to a unit reduction in net surplus is given by $\frac{\bar{\gamma}_w(w')\bar{\gamma}^{\mathfrak{s}}_w(w')}{\bar{\gamma}_w(w')+\bar{\gamma}^{\mathfrak{s}}_w(w')}$, to which we refer as the match sensitivity. The match sensitivity is the common rate at which the two spouses leave (or enter) the match once the reduction is split between them according to their bargaining weights. Higher matching elasticities imply higher sensitivity and stronger sorting responses to changes in net surplus.

To solve these equations, use equation (15) to substitute for $\dot{\pi}$ in (16). One obtains a linear integral equation that determines $\dot{u}$ as a function of $\dot{v}$, which can be written as

$$\dot{u} = \mathbf{\Theta}\dot{v} + \mathbf{A}\dot{u} - \mathbf{B}\dot{u}, \tag{18}$$

where the linear operators $\mathbf{\Theta}$, $\mathbf{A}$, and $\mathbf{B}$ are defined solely by $\gamma$.[12] One can verify that both $\mathbf{A}$ and $\mathbf{B}$ are positive and that $\mathbf{A}+\mathbf{B}$ is Markov, i.e., a stochastic matrix in the finite-type case. This implies

[12] In this expression, the $(w,w')$ element of $\mathbf{\Theta}\dot{v}$ is given by $(\mathbf{\Theta}\dot{v})(w,w') = \int \mathbf{\Theta}_{w,w'}(\tilde{w},\tilde{w}')\dot{v}(\tilde{w},\tilde{w}')d(\tilde{w},\tilde{w}')$ where $\mathbf{\Theta}_{w,w'}(\tilde{w},\tilde{w}') = \frac{\bar{\gamma}^{\mathfrak{s}}_w(w')}{\bar{\gamma}_w(w')+\bar{\gamma}^{\mathfrak{s}}_w(w')}\delta_{\tilde{w}=w}\delta_{\tilde{w}'=w'}$ and $\delta_{\tilde{w}=w}$ is the Dirac delta function that puts all mass on the point $w$. The terms $\mathbf{A}\dot{u}$ and $\mathbf{B}\dot{u}$ have an analogous structure, with $\mathbf{A}_{w,w'}(\tilde{w},\tilde{w}') = \frac{\gamma_w(\tilde{w}'|w')}{\bar{\gamma}_w(w')+\bar{\gamma}^{\mathfrak{s}}_w(w')}\delta_{\tilde{w}=w}$ and $\mathbf{B}_{w,w'}(\tilde{w},\tilde{w}') =$

that the spectral radius of $\mathbf{A}-\mathbf{B}$ is strictly less than one (see the appendix for details), which allows us to write the solution as

$$\dot{u}=\mathbf{K}\dot{v} \tag{19}$$

where $\mathbf{K}$ is given by

$$\mathbf{K}:=(\mathbf{I}-\mathbf{A}+\mathbf{B})^{-1}\mathbf{\Theta}=\mathbf{\Theta}+(\mathbf{A}-\mathbf{B})\mathbf{\Theta}+(\mathbf{A}-\mathbf{B})^{2}\mathbf{\Theta}+\cdots. \tag{20}$$

The geometric series in (20) has a simple economic interpretation. The first term, $\mathbf{\Theta}$, captures how agents within couples share the change in family surplus $\dot{v}$ according to their bargaining weights encapsulated in $\mathbf{\Theta}$. This sharing changes the outside options of spouses. The second term, $(\mathbf{A}-\mathbf{B})\mathbf{\Theta}$, describes how spouses adjust consumption shares in response to these changes in outside options. These adjustments in turn further change outside options, which requires further adjustments $(\mathbf{A}-\mathbf{B})^{2}\mathbf{\Theta}$, and so on. Since the spectral radius of $\mathbf{A}-\mathbf{B}$ is less than one, each round of adjustments is smaller than the previous one, and the series converges.

We can now combine (19) and (13) to find the sharing responses $\dot{u}$ as a function of $\dot{\mathcal{T}}$. Substituting this solution into (17), we can then find $\dot{q}/q$ and, therefore, $\dot{f}/f$. We summarize these results in the following lemma.

**Lemma 1.** *Let the operator* $\mathbf{M}$ *be defined as* $\mathbf{M}:=\mathbf{CK}$*, where the operator* $\mathbf{C}$ *is given by* $\mathbf{C}_{w,w'}(\tilde{w},\tilde{w}'):=\bar{\gamma}_{w}(w')\delta_{\tilde{w}=w}\delta_{\tilde{w}'=w'}-\gamma_{w}(\tilde{w}'|w')\delta_{\tilde{w}=w}$*. The sorting and sharing responses to any tax reform are given by*

$$\dot{u}=-\mathbf{K}\dot{\mathcal{T}},\quad \dot{f}/f=\dot{q}/q=-\mathbf{M}\dot{\mathcal{T}}. \tag{21}$$

Equation (21) characterizes the sharing and sorting responses to arbitrary tax reforms. These responses are captured by the operators $\mathbf{K}$ and $\mathbf{M}$, which depend solely on matching elasticities. The operator $\mathbf{K}$ captures the economic incidence of tax reforms: it takes the statutory changes in taxes $\dot{\mathcal{T}}$ for all couples and returns the changes in the pecuniary utility of every agent, $-\mathbf{K}\dot{\mathcal{T}}$. The operator $\mathbf{C}$ translates changes in pecuniary utilities into changes in choice probabilities, and the operator $\mathbf{M}$ combines the two steps.

In Lemma 1, we constructed particular operators $\mathbf{K}$ and $\mathbf{M}$ that describe sorting and sharing responses. These operators are not unique. Tax reforms $\dot{\mathcal{T}}$ are symmetric functions of $(w,w')$, so (21) determines how $\mathbf{K}$ and $\mathbf{M}$ act on symmetric functions but leaves their action on all other functions unrestricted. Any $\hat{\mathbf{K}}$ and $\hat{\mathbf{M}}$ that agree with $\mathbf{K}$ and $\mathbf{M}$ on symmetric functions therefore satisfy (21) as well. For the purpose of theoretical analysis, all such operators are equivalent: they describe the same

$\frac{\gamma_{w'}(\tilde{w}'|w)}{\bar{\gamma}_{w}(w')+\bar{\gamma}^{\mathbf{s}}_{w}(w')}\delta_{\tilde{w}=w'}$. Equations such as (18) are known as Fredholm integral equations of the second kind. The role of these equations in the theory of taxation was first highlighted by Sachs et al. (2020). In the case of a finite number of types, $\mathbf{\Theta}$, $\mathbf{A}$, $\mathbf{B}$ are square matrices.

equilibrium responses in terms of the same matching elasticities. The advantage of the construction of $\mathbf{K}$ and $\mathbf{M}$ that we provide in the text is that it is general and applies to any law of shocks $\Psi$. We provide illustrations of other equivalent operators in Section 6 and in the appendix.

## Illustration in the two-type economy

We illustrate applications of Lemma 1 using the two-type economy introduced in Section 3. We arrange the sharing responses in the vector $\dot{u}$ as $\dot{u} = [\dot{u}_H(H), \dot{u}_H(L), \dot{u}_L(H), \dot{u}_L(L)]^{\mathrm{T}}$. Direct calculations show that $\mathbf{\Theta}$, $\mathbf{A}$ and $\mathbf{B}$ can be written as

$$\mathbf{\Theta} = \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1-\theta & 0 & 0 \\ 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} 0 & 1/2 & 0 & 0 \\ \theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\theta \\ 0 & 0 & 1/2 & 0 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1-\theta \\ \theta & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \end{bmatrix},$$

where $\theta = \frac{\bar{\gamma}_H(L)}{\bar{\gamma}_L(H)+\bar{\gamma}_H(L)}$ is the bargaining weight of $L$ in $HL$ matches.[13] Using (20) to compute $\mathbf{K}$, we obtain

$$\mathbf{K} = \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ \theta/2 & 1-\theta & 0 & -(1-\theta)/2 \\ -\theta/2 & 0 & \theta & (1-\theta)/2 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}.$$

One can verify that $(\mathbf{A}-\mathbf{B})^k = 0$ for all $k > 1$, which shows that with two types all equilibrium adjustments described by the geometric series in equation (20) happen in a single round.

Substituting this $\mathbf{K}$ into (21), we obtain $\dot{u}$. Spouses in homogeneous matches share the tax burden equally, $\dot{u}_L(L) = -\frac{1}{2}\dot{\mathcal{T}}(L,L)$ and $\dot{u}_H(H) = -\frac{1}{2}\dot{\mathcal{T}}(H,H)$. The effect of the tax reform on spouses in heterogeneous matches is

$$\dot{u}_L(H) = -\frac{1}{2}\dot{\mathcal{T}}(L,L) + \frac{\theta}{2}\dot{\varphi}, \quad \dot{u}_H(L) = -\frac{1}{2}\dot{\mathcal{T}}(H,H) + \frac{1-\theta}{2}\dot{\varphi}, \tag{22}$$

where

$$\dot{\varphi} = \dot{\mathcal{T}}(H,H) + \dot{\mathcal{T}}(L,L) - \dot{\mathcal{T}}(L,H) - \dot{\mathcal{T}}(H,L)$$

captures the change in the modularity of taxes induced by the reform. This equation illustrates the point that we made in equation (16) that sharing is determined by two motives: changes in outside options and changes in net surplus. Consider a person $L$ in a heterogeneous match. The outside option of that person is to marry an $L$ type. The tax reform changes the value of that outside option by $\dot{\pi}_L(H) = -\frac{1}{2}\dot{\mathcal{T}}(L,L)$. The net surplus is equal to the difference between the surplus in heterogeneous matches $v(H,L) = v(L,H)$ and the two outside options of spouses, $\frac{1}{2}v(L,L)$ and $\frac{1}{2}v(H,H)$. Therefore, changes in the net surplus are captured by changes in the modularity of $v$. If a tax reform makes the tax schedule

[13] With two types, we have $\bar{\gamma}_w(w') = \gamma_w(\tilde{w}'|w')$ for all $w$, $w' \neq \tilde{w}'$, so that $\mathbf{\Theta}$, $\mathbf{A}$ and $\mathbf{B}$ can be expressed in terms of bargaining weights. The bargaining weights in homogeneous matches are $\frac{1}{2}$, while the bargaining weights in heterogeneous matches are $\theta$ for $L$ and $1-\theta$ for $H$.

more supermodular, then $\dot{\varphi} > 0$. More supermodular taxes make after-tax earnings less supermodular in productivities and, therefore, reduce the benefits of assortative matching. Consequently, the net family surplus in heterogeneous matches increases, and the $L$-type captures a share $\theta$ of that surplus change. Similar intuition applies to the expression for $\dot{u}_H(L)$.

The modularity change $\dot{\varphi}$ also affects sorting. Applying (21) to find $\dot{f}/f$, we obtain

$$\frac{\dot{f}(L,H)}{f(L,H)} = \frac{\dot{f}(H,L)}{f(H,L)} = \frac{\bar{\gamma}_L(H)\bar{\gamma}_H(L)}{\bar{\gamma}_L(H) + \bar{\gamma}_H(L)} \frac{\dot{\varphi}}{2}, \tag{23}$$

which shows that the share of heterogeneous matches increases if taxes become more supermodular. The share of homogeneous matches falls by the same amount.

In Figure 1(a), we considered a tax reform that increases individual-based taxes for the highest earners. Under positive complementarities, this reform increases taxes for $HH$ families by $\dot{\mathcal{T}}(H,H) = 2$ and has no effect on taxes of other families. Consequently, equation (22) yields

$$\dot{u}_H(H) = -1, \quad \dot{u}_H(L) = -\theta, \quad \dot{u}_L(H) = \theta, \quad \dot{u}_L(L) = 0.$$

Under negative complementarities, this reform increases taxes for $HL$ families by $\dot{\mathcal{T}}(H,L) = \dot{\mathcal{T}}(L,H) = 1$, and equation (22) implies

$$\dot{u}_H(H) = 0, \quad \dot{u}_H(L) = -1 + \theta, \quad \dot{u}_L(H) = -\theta, \quad \dot{u}_L(L) = 0.$$

As we discussed in Section 3, in both cases, consumption sharing increases in heterogeneous matches. The bargaining weight $\theta = \frac{\bar{\gamma}_H(L)}{\bar{\gamma}_L(H)+\bar{\gamma}_H(L)}$ captures the strength of this sharing response.

## When do taxes not affect sorting?

Equation (23) shows that in the two-type example a tax reform does not affect equilibrium sorting if it leaves the modularity of taxes unchanged, $\dot{\varphi} = 0$. This result is more general and applies to economies with any number of types. The following lemma formalizes this observation.

**Lemma 2.** *$\dot{f} = 0$ if and only if $\dot{\mathcal{T}}$ is additively separable in the sense that $\dot{\mathcal{T}}(w,w') = \dot{\tau}(w) + \dot{\tau}(w')$ for some $\dot{\tau}$. In particular, there are no sorting responses if*

*(a). the tax reform is a uniform lump-sum tax or transfer, $\dot{\mathcal{T}} = \pm 1$, in which case sharing responses are $\dot{u} = \mp\frac{1}{2}$;*

*(b). there are no complementarities and taxes are individual-based, in which case $\dot{u} = -\dot{T}$, where $\dot{T}$ is the perturbation of the individual earnings tax schedule.*

This lemma makes several observations. As has been known in the matching literature since Becker (1973), equilibrium sorting is determined by the modularity of the surplus function $v$, which is captured

by the cross-partial derivative of $v$. Since $\dot{v} = -\dot{\mathcal{T}}$, the modularity of $v$ remains unchanged if $\dot{\mathcal{T}}$ is additively separable in the two spouses' types. Lemma 2 also provides two important special cases of such reforms. First, a uniform lump-sum tax or transfer to all families has no effect on equilibrium sorting. All families share the burden of such taxes equally, and an extra dollar of taxes for all couples reduces spousal consumption by 50 cents. Second, if there are no complementarities and taxes are individual-based, then tax reforms have no sorting effects, and their statutory and economic incidences coincide, $\dot{u} = -\dot{T}$. Conversely, complementarities in labor supply, induced either through home production or through jointness of the tax function, lead to non-trivial sorting and sharing effects of tax reforms.

## 4.2 Behavioral effects

The analysis of the behavioral effects closely parallels that of behavioral effects in multi-good Mirrleesian economies. The surplus maximization problem (5) that describes optimal earnings choices within matches is mathematically isomorphic to a problem of a single consumer who chooses an optimal multi-good consumption bundle subject to a general, potentially non-linear and jointed tax function. Similarly to the previous section, we use the shorthand $\frac{\partial}{\partial y}\mathcal{T}(w,w')$ to denote the marginal tax rate $\frac{\partial}{\partial y}\mathcal{T}(y_w(w'), y_w^{\mathfrak{s}}(w'))$ of type $w$ married to $w'$, and $\frac{\partial}{\partial y}\mathcal{T}^{\mathfrak{s}}(w,w')$ to denote the marginal tax rate of their spouse. The same convention applies to $\frac{\partial}{\partial y}\dot{\mathcal{T}}$ and $\frac{\partial}{\partial y}\dot{\mathcal{T}}^{\mathfrak{s}}$, which denote the statutory changes in marginal taxes induced by the reform.

**Lemma 3.** *The behavioral responses $\dot{y}$ to any tax reform are summarized by $\xi \equiv (\xi_w(w'))_{w,w'}$ and $\zeta \equiv (\zeta_w(w'))_{w,w'}$, which may depend on $\mathcal{T}$ but are independent of the reform, so that*

$$\frac{\dot{y}_w(w')}{y_w(w')} = -\frac{\xi_w(w')}{1-\frac{\partial}{\partial y}\mathcal{T}(w,w')}\frac{\partial}{\partial y}\dot{\mathcal{T}}(w,w') - \frac{\zeta_w(w')}{1-\frac{\partial}{\partial y}\mathcal{T}^{\mathfrak{s}}(w,w')}\frac{\partial}{\partial y}\dot{\mathcal{T}}^{\mathfrak{s}}(w,w'). \tag{24}$$

*The variable $\xi_w(w')$ is positive, while the sign of $\zeta_w(w')$ is the same as the sign of $\frac{l'\frac{\partial^2}{\partial l\partial l'}\mathcal{N}}{-\frac{\partial}{\partial l}\mathcal{N}} - \frac{y'\frac{\partial^2}{\partial y\partial y'}\mathcal{T}}{1-\frac{\partial}{\partial y}\mathcal{T}}$ evaluated at the optimal choices $(y_w(w'), y_w^{\mathfrak{s}}(w'))$. In addition, $\zeta_w(w')$ and $\zeta_w^{\mathfrak{s}}(w')$ satisfy Slutsky symmetry, i.e.,*

$$\frac{y_w(w')\zeta_w(w')}{1-\frac{\partial}{\partial y}\mathcal{T}^{\mathfrak{s}}(w,w')} = \frac{y_w^{\mathfrak{s}}(w')\zeta_w^{\mathfrak{s}}(w')}{1-\frac{\partial}{\partial y}\mathcal{T}(w,w')}.$$

The variable $\xi_w(w')$ in Lemma 3 is the elasticity of labor supply of person $w$ married to $w'$ with respect to their retention rate, and the variable $\zeta_w(w')$ is their labor supply elasticity with respect to the retention rate of the spouse. These two variables are analogues of compensated own- and cross-price elasticities in consumer theory. The own elasticity is positive, while the sign of the cross-elasticity depends on cross partials of $\mathcal{N}$ and $\mathcal{T}$. A positive value of $\zeta_w(w')$ means that the individual reduces their earnings in response to an increase in their partner's marginal tax rates. This occurs either if there are positive complementarities or negative jointness in the tax function.

### 4.3 Effects of tax reforms on revenues and welfare

Lemmas 1 and 3 allow us to characterize the impact of any tax reform on government revenues and social welfare. The government tax revenues are

$$R := \frac{1}{2}\int \mathcal{T}(y, y^{\mathfrak{s}})dF,$$

where the factor $\frac{1}{2}$ appears in front of the integral to avoid double counting of taxes since the mass of couples is equal to $\frac{1}{2}$. Differentiating this expression in the direction $\dot{\mathcal{T}}$, while taking into account that $\mathcal{T}$, $y$, $y^{\mathfrak{s}}$ and $f$ are all affected by the reform, and applying Lemmas 1 and 3, we obtain

$$\begin{aligned}\dot{R} &= \frac{1}{2}\int \dot{\mathcal{T}}dF + \frac{1}{2}\int \frac{\partial \mathcal{T}}{\partial y}\dot{y}dF + \frac{1}{2}\int \frac{\partial \mathcal{T}^{\mathfrak{s}}}{\partial y}\dot{y}^{\mathfrak{s}}dF + \frac{1}{2}\int \mathcal{T}\frac{\dot{f}}{f}dF \\ &= \underbrace{\frac{1}{2}\int \dot{\mathcal{T}}dF}_{\text{mechanical}} - \underbrace{\int \frac{y\xi\frac{\partial \mathcal{T}}{\partial y}}{1-\frac{\partial \mathcal{T}}{\partial y}}\frac{\partial \dot{\mathcal{T}}}{\partial y}dF}_{\text{direct behavioral}} - \underbrace{\int \frac{y\zeta\frac{\partial \mathcal{T}^{\mathfrak{s}}}{\partial y}}{1-\frac{\partial \mathcal{T}^{\mathfrak{s}}}{\partial y}}\frac{\partial \dot{\mathcal{T}}}{\partial y}dF}_{\text{indirect behavioral}} - \underbrace{\frac{1}{2}\int \mathcal{T}\cdot\mathbf{M}\dot{\mathcal{T}}dF}_{\text{sorting}}.\end{aligned} \tag{25}$$

The four terms in this equation represent the mechanical, direct and indirect behavioral, and sorting effects of tax reforms. The mechanical and behavioral effects have the structure familiar from Mirrleesian models. The sorting effect is new and is captured by the operator $\mathbf{M}$ constructed from matching elasticities. The sorting effect can equivalently be written as

$$\frac{1}{2}\int \mathbf{M}^{*}\mathcal{T}\cdot\dot{\mathcal{T}}dF, \tag{26}$$

where the integral operator $\mathbf{M}^{*}$ is related to $\mathbf{M}$ via $\mathbf{M}^{*}_{w,w'}(\tilde{w},\tilde{w}') = \mathbf{M}_{\tilde{w},\tilde{w}'}(w,w')\frac{f(\tilde{w},\tilde{w}')}{f(w,w')}$.[14] This alternative representation will be more convenient in some of the applications.

The impact of tax reforms on welfare can be found similarly. For concreteness, suppose that social welfare is given by

$$W := \int \alpha U_w dG, \tag{27}$$

where $\alpha \equiv (\alpha(w))_w$ are Pareto weights, i.e., a positive function normalized to $\int \alpha dG = 1$. Using (9) to compute the derivatives of $U_w$, we obtain

$$\dot{W} = \int \alpha(w)\dot{u}_w(w')\underbrace{q_w(w')g(w)}_{=f(w,w')}d(w,w') = -\int \alpha\cdot\mathbf{K}\dot{\mathcal{T}}dF = -\underbrace{\int \mathbf{K}^{*}\alpha\cdot\dot{\mathcal{T}}dF}_{\text{sharing}}, \tag{28}$$

[14] To see this, write the integral operation of sorting effects explicitly and change the order of integration

$$\begin{aligned}\int \mathcal{T}\cdot\mathbf{M}\dot{\mathcal{T}}dF &= \int \mathcal{T}(w,w')\left[\int \mathbf{M}_{w,w'}(\tilde{w},\tilde{w}')\dot{\mathcal{T}}(\tilde{w},\tilde{w}')d(\tilde{w},\tilde{w}')\right]f(w,w')d(w,w') \\ &= \int\left[\int \mathbf{M}_{w,w'}(\tilde{w},\tilde{w}')\frac{f(w,w')}{f(\tilde{w},\tilde{w}')}\mathcal{T}(w,w')d(w,w')\right]\dot{\mathcal{T}}(\tilde{w},\tilde{w}')f(\tilde{w},\tilde{w}')d(\tilde{w},\tilde{w}') \\ &= \int\left[\int \mathbf{M}^{*}_{w,w'}(\tilde{w},\tilde{w}')\mathcal{T}(\tilde{w},\tilde{w}')d(\tilde{w},\tilde{w}')\right]\dot{\mathcal{T}}(w,w')f(w,w')d(w,w') = \int \mathbf{M}^{*}\mathcal{T}\cdot\dot{\mathcal{T}}dF.\end{aligned}$$

where the operator $\mathbf{K}^*$ is defined analogously to $\mathbf{M}^*$. By the envelope theorem, changes in sorting have no first-order effect on welfare; the reform affects welfare only through the changes in pecuniary utilities $\dot{u}$, and this equation shows how endogenous sharing determines this effect.

Using these two expressions for $\dot{R}$ and $\dot{W}$, one can describe the incidence of any tax reform, test whether existing taxes admit Pareto-improving reforms, and characterize taxes that maximize the social welfare function. In the next section, we apply these formulas to study properties of the two most common forms of taxation of married individuals: family- and individual-based taxes.

# 5 Individual- and family-based taxation of income

We start with the analysis of family-based taxation because it is somewhat simpler and allows us to present the key steps more transparently.

## 5.1 Family-based taxation

Under family-based taxation, taxes satisfy $\mathcal{T}(y_w(w'), y_w^{\mathfrak{s}}(w')) = T^{\mathrm{F}}(y_w(w') + y_w^{\mathfrak{s}}(w'))$, or $\mathcal{T} = T^{\mathrm{F}}$ for short, and admissible tax reforms are of the form $\dot{\mathcal{T}} = \dot{T}^{\mathrm{F}}$. Let

$$Y(w, w') := y_w(w') + y_w^{\mathfrak{s}}(w')$$

be family earnings under the tax schedule $T^{\mathrm{F}}$. We use $H^{\mathrm{F}}$ to denote the distribution of family earnings, and we use $h^{\mathrm{F}}$ to denote its density. Both spouses face the same marginal tax rate, which we denote by $\frac{\partial T^{\mathrm{F}}}{\partial Y}$.

Substituting $T^{\mathrm{F}}$ and $\dot{T}^{\mathrm{F}}$ for $\mathcal{T}$ and $\dot{\mathcal{T}}$ into equation (25), we can express the effect of family-based tax reforms on revenues as

$$\dot{R} = \frac{1}{2}\int \left(1 - \mathbb{E}[\mathbf{M}^* T^{\mathrm{F}} | Y]\right) \dot{T}^{\mathrm{F}} dH^{\mathrm{F}} - \frac{1}{2}\int \frac{Y \bar{\xi}^{\mathrm{F}} \frac{\partial T^{\mathrm{F}}}{\partial Y}}{1 - \frac{\partial T^{\mathrm{F}}}{\partial Y}} \frac{\partial}{\partial Y} \dot{T}^{\mathrm{F}} dH^{\mathrm{F}}, \tag{29}$$

where $\bar{\xi}^{\mathrm{F}} := 2\mathbb{E}\left[\frac{y}{Y}(\xi + \zeta) | Y\right]$ is the elasticity of family earnings, a weighted average across the two spouses of the sum of their own- and cross-elasticities of labor supply. Equations similar to (29) appear in Mirrleesian models, with the key difference that the first integral captures the mechanical effect and also includes the sorting effect, through the operator $\mathbf{M}^*$.

For concreteness, take a reform that raises the marginal tax rate at the family earnings level $t$, so that the tax bill of all families with earnings above $t$ goes up by one dollar, $\dot{T}^{\mathrm{F}} = 1_{Y>t}$.[15] By (29), this reform changes revenues by $\frac{1}{2}\Delta^{\mathrm{F}}(t)$, where

$$\Delta^{\mathrm{F}}(t) := (1 - H^{\mathrm{F}}(t))\left(1 - \mathbb{E}[\mathbf{M}^* T^{\mathrm{F}} | Y > t]\right) - \frac{t \bar{\xi}^{\mathrm{F}}(t) \frac{\partial}{\partial Y} T^{\mathrm{F}}(t)}{1 - \frac{\partial}{\partial Y} T^{\mathrm{F}}(t)} h^{\mathrm{F}}(t). \tag{30}$$

[15] This reform is not twice continuously differentiable: it raises the marginal tax rate by $\frac{\partial \dot{T}^{\mathrm{F}}}{\partial Y} = \delta_{Y=t}$. However, it is the pointwise limit of twice continuously differentiable reforms, and its effects on revenues and utilities are the limits of their effects; see the appendix.

Thus, the sign of $\Delta^{\mathrm{F}}(t)$ tells us whether the marginal tax rate at $t$ is to the left or to the right of the local peak of the Laffer curve.

**Welfare-maximizing taxes**

Consider the problem of choosing a family-based tax schedule $T^{\mathrm{F}}$ that maximizes welfare $W$ subject to the budget constraint $R \geq 0$. By Lemma 2, a unit increase in lump-sum taxes reduces the utilities of all agents, and therefore social welfare, by $\frac{1}{2}$. Since this reform has no behavioral or sorting effects, it also raises revenues by $\frac{1}{2}$. Therefore, the Lagrange multiplier on the budget constraint equals 1, and the optimal family-based taxes $T^{\mathrm{F}}$ satisfy

$$\dot{W} + \dot{R} = 0 \quad \text{for all } \dot{T}^{\mathrm{F}}. \tag{31}$$

By (28), raising the marginal tax rate at $t$ reduces welfare by $(1-H^{\mathrm{F}}(t))\mathbb{E}[\mathbf{K}^*\alpha|Y>t]$, while it changes revenues by $\frac{1}{2}\Delta^{\mathrm{F}}(t)$. Hence, by (31), optimal taxes satisfy, at every family earnings level $t$,

$$(1-H^{\mathrm{F}}(t))\mathbb{E}[\mathbf{K}^*\alpha|Y>t] = \frac{1}{2}\Delta^{\mathrm{F}}(t). \tag{32}$$

Condition (32) states that, at the optimum, the welfare loss from raising the marginal tax rate at $t$ equals the resulting change in revenues. Substituting (30) into (32) yields the following characterization of optimal taxes.

**Proposition 1.** *The optimal family-based tax schedule, at every family earnings level $t$, satisfies*

$$\frac{\partial T^{\mathrm{F}}(t)/\partial Y}{1-\partial T^{\mathrm{F}}(t)/\partial Y} = \frac{1}{\bar{\xi}^{\mathrm{F}}(t)}\frac{1-H^{\mathrm{F}}(t)}{h^{\mathrm{F}}(t)t}\mathbb{E}\left[1-2\mathbf{K}^*\alpha-\mathbf{M}^*T^{\mathrm{F}}|Y>t\right]. \tag{33}$$

The optimal family-based tax formula has the flavor of the "ABC" formulas popularized in Mirrleesian economies by Diamond (1998) and Saez (2001). As in the ABC formula, the optimal marginal tax rate at the earnings level $t$ is proportional to the inverse of the elasticity of earnings $\bar{\xi}^{\mathrm{F}}(t)$ (the "A" term in Diamond's formula) and to the earnings hazard ratio $\frac{1-H^{\mathrm{F}}(t)}{h^{\mathrm{F}}(t)t}$ (the "C" term). The "B" term, however, is different. It includes the sorting and sharing effects, captured by the operators $\mathbf{K}^*$ and $\mathbf{M}^*$, both of which are constructed from matching elasticities $\gamma$.

**Efficient taxes**

Optimal taxation requires taking a stance on the social weights in the welfare function. One may ask a weaker question: whether it is possible to reform a given tax $T^{\mathrm{F}}$ in a budget-feasible way so as to make all individuals better off.[16] We say that taxes are (locally) *efficient* if there is no budget-feasible

[16] A number of recent papers (e.g., Bierbrauer et al. (2023), Bergstrom and Dodds (2026), Becko and Sztutman (2026)) design tests of the (local) efficiency of a given tax schedule in Mirrleesian settings. Such tests determine whether it is possible to perturb $\mathcal{T}$ in a revenue-neutral way so as to make all agents better off. A closely related "inverse Pareto weight" literature (Blundell et al. (2009), Lockwood and Weinzierl (2016), Hendren (2020)) seeks the Pareto weights under which the existing tax $T$ is a (local) optimum. One can show that $T$ is efficient if and only if the implied inverse Pareto weights are nonnegative. Following most of the literature, we restrict attention to the local notion of efficiency in this section. See Werning (2007) for the results on global efficiency.

perturbation, $\dot{R} \geq 0$, that makes agents in all matches better off, $\dot{u} > 0$.

**Proposition 2.** *The tax $T^{\mathrm{F}}$ is efficient if and only if there exist Pareto weights $\alpha \equiv (\alpha(w, w'))_{w,w'}$ normalized to one*[17] *such that (32), or equivalently (33), holds at every family earnings level $t$.*

Proposition 2 is the local version of the familiar result that an allocation is Pareto efficient if and only if it maximizes a weighted sum of utilities for some Pareto weights. It thus extends the inverse-optimum approach discussed in footnote 16 to Beckerian environments. Unlike (27), the Pareto weights may differ across agents of the same type married to different partners, since our notion of efficiency requires agents in all matches to be better off.[18]

Suppose that raising the marginal tax rate at $t$ lowered the utility of each spouse in families with earnings above $t$ by half a dollar and did not affect anyone else, as in Mirrleesian models, so that $\dot{u} = -\mathbf{K}1_{Y>t} = -\frac{1}{2}1_{Y>t}$. The left-hand side of (32) would then equal $\frac{1}{2}\int_{Y>t} \alpha dF$, and (32) would state that the total Pareto weight on agents in families with earnings above $t$ equals $\Delta^{\mathrm{F}}(t)$. The Laffer curve would pin down the implied weights, and taxes would be efficient if and only if $\Delta^{\mathrm{F}}$ is nonincreasing and takes values between zero and one. In particular, taxes would be inefficient whenever the marginal tax rate at some $t$ is to the right of the peak of the Laffer curve, $\Delta^{\mathrm{F}}(t) < 0$.

In Beckerian environments, this is no longer the case, because of the sharing effects of the reform on other families. For example, in the two-type economy of Section 4, a tax increase on $HH$ couples lowers the outside option of $H$ types married to $L$ types, and by (22) their $L$ spouses gain, $\dot{u}_L(H) = \theta/2 > 0$. A planner who puts a sufficient weight on $L$ types may then optimally set the marginal tax rate beyond the peak of the Laffer curve. Instead, Proposition 2 implies that taxes are inefficient if two conditions hold at some family earnings level $t$. First, the marginal tax rate at $t$ is to the right of the peak of the Laffer curve, $\Delta^{\mathrm{F}}(t) < 0$. Second, raising it makes no agent better off, $\mathbf{K}1_{Y>t} \geq 0$. In this case, the left-hand side of (32) is nonnegative for any Pareto weights, while the right-hand side is negative. Intuitively, any tax increase makes some agents worse off, so Pareto improvements must come from tax cuts. Cutting the marginal tax rate at $t$ raises revenues without making any agent worse off, and rebating these revenues lump-sum makes all agents better off.

Proposition 2 also gives a practical way to test whether an existing tax schedule is efficient. Combining estimates of behavioral and matching elasticities, the distribution of family earnings, and marginal tax rates, one can compute the right-hand side of (32) on a fine grid of family earnings. On a discretized productivity grid, $\mathbf{K}^*$ is a matrix, so the term $\mathbb{E}[\mathbf{K}^*\alpha|Y > t]$ is linear in $\alpha$, so that (32) becomes a system of linear equations with one equation for each earnings level on the grid. Then, a standard

[17] In general, these weights may have to be a probability measure over agents in matches rather than a nonnegative function; see the appendix.

[18] One may instead use an interim notion of efficiency: taxes are interim (locally) efficient if there is no budget-feasible perturbation that makes each agent better off before entering the marriage market, $\dot{U}_w > 0$. Proposition 2 continues to characterize this notion of efficiency with the added restriction that $\alpha$ is independent of the partner's productivity $w'$.

linear programming solver can be used to efficiently determine whether this system has a solution with nonnegative weights normalized to one: if it does, taxes are efficient; if it does not, they are inefficient. The exact same idea applies to Mirrleesian settings; however, there efficiency can be tested by directly examining $\Delta^{\mathrm{F}}$. Since the sorting and sharing effects are absent, the left-hand side of (32) equals $\frac{1}{2}(1-H^{\mathrm{F}}(t))\mathbb{E}[\alpha|Y>t]$, and so by Proposition 2, taxes are efficient if and only if $\Delta^{\mathrm{F}}$ is a nonincreasing function bounded between zero and one, which is precisely the characterization of Bierbrauer et al. (2023).

## 5.2 Individual-based taxation

Under individual-based taxation, taxes take the form $\mathcal{T}(y_w(w'), y_w^{\mathfrak{s}}(w')) = T(y_w(w')) + T(y_w^{\mathfrak{s}}(w'))$, or $\mathcal{T} = T + T^{\mathfrak{s}}$ for short, and admissible tax reforms are of the form $\dot{\mathcal{T}} = \dot{T} + \dot{T}^{\mathfrak{s}}$. We use $H$ to denote the distribution of individual earnings under the tax schedule $T$, and we use $h$ to denote its density.

Similarly to the previous section, we study optimal and efficient taxes using a reform that raises the marginal tax rate at the individual earnings level $t$, which now increases the tax bill of a couple by $\dot{\mathcal{T}} = 1_{y>t} + 1_{y^{\mathfrak{s}}>t}$ (see footnote 15). Following the same steps as before, we can obtain that such a reform changes revenues by $\Delta(t)$ given by

$$\begin{aligned}\Delta(t) :=&(1-H(t))\left(1-\mathbb{E}\left[\frac{1}{2}\mathbf{M}^*(T+T^{\mathfrak{s}})+\frac{1}{2}(\mathbf{M}^*(T+T^{\mathfrak{s}}))^{\mathfrak{s}}|y>t\right]\right)\\ &-t\left(\bar{\xi}(t)\frac{\partial T(t)/\partial y}{1-\partial T(t)/\partial y}+\mathbb{E}\left[\zeta\frac{\partial T^{\mathfrak{s}}/\partial y}{1-\partial T^{\mathfrak{s}}/\partial y}|y=t\right]\right)h(t),\end{aligned} \tag{34}$$

where $\bar{\xi} := \mathbb{E}[\xi|y]$ is the average elasticity of individual earnings. The first term combines the mechanical effect and the sorting effect, which now has two terms, $\mathbf{M}^*(T+T^{\mathfrak{s}})$ and $(\mathbf{M}^*(T+T^{\mathfrak{s}}))^{\mathfrak{s}}$, one for each spouse.[19] The second term is the behavioral effect. In addition to the direct effect, which is proportional to the own elasticity $\bar{\xi}(t)$, it includes the indirect effect. The spouses of individuals with earnings $t$ adjust their labor supply, which affects revenues in proportion to the cross-elasticity $\zeta$ and the spouses' marginal tax rates.

**Welfare-maximizing taxes**

As in Section 5.1, the Lagrange multiplier on the budget constraint equals 1. By (28), raising the marginal tax rate at $t$ reduces welfare by $(1-H(t))\mathbb{E}[\mathbf{K}^*\alpha + (\mathbf{K}^*\alpha)^{\mathfrak{s}}|y>t]$, while it changes revenues by $\Delta(t)$. Hence, optimal taxes satisfy, at every individual earnings level $t$,

$$(1-H(t))\mathbb{E}[\mathbf{K}^*\alpha + (\mathbf{K}^*\alpha)^{\mathfrak{s}}|y>t] = \Delta(t). \tag{35}$$

Substituting (34) into (35) yields the following characterization of optimal taxes.

[19] To derive equation (34), we used the symmetry of $F$, so that $\int \mathbf{M}^*(T+T^{\mathfrak{s}})\cdot\dot{T}^{\mathfrak{s}}dF = \int(\mathbf{M}^*(T+T^{\mathfrak{s}}))^{\mathfrak{s}}\cdot\dot{T}dF$. In general, $\mathbf{M}^*(T+T^{\mathfrak{s}})$ is not symmetric.

**Proposition 3.** *The optimal individual-based tax schedule, at every individual earnings level $t$, satisfies*

$$\frac{\partial T(t)/\partial y}{1-\partial T(t)/\partial y} = \frac{1}{\bar{\xi}(t)}\frac{1-H(t)}{h(t)t}\mathbb{E}\left[1-\mathbf{K}^*\alpha-(\mathbf{K}^*\alpha)^{\mathfrak{s}}-\frac{1}{2}\mathbf{M}^*(T+T^{\mathfrak{s}})-\frac{1}{2}(\mathbf{M}^*(T+T^{\mathfrak{s}}))^{\mathfrak{s}}|y>t\right] - \mathbb{E}\left[\frac{\zeta}{\bar{\xi}(t)}\frac{\partial T^{\mathfrak{s}}/\partial y}{1-\partial T^{\mathfrak{s}}/\partial y}|y=t\right]. \tag{36}$$

**Efficient taxes**

The analogue of Proposition 2 is as follows.

**Proposition 4.** *The tax $T$ is efficient if and only if there exist Pareto weights $\alpha \equiv (\alpha(w,w'))_{w,w'}$ normalized to one such that (35), or equivalently (36), holds at every individual earnings level $t$.*

In particular, taxes are inefficient if two conditions hold at some individual earnings level $t$. First, the marginal tax rate at $t$ is to the right of the peak of the Laffer curve, $\Delta(t) < 0$. Second, raising it makes no agent better off, $\mathbf{K}(1_{y>t} + 1_{y^{\mathfrak{s}}>t}) \geq 0$. Without complementarities, the second condition always holds. By Lemma 2, raising the marginal tax rate at $t$ then reduces the utility of each individual with earnings above $t$ by one dollar and does not affect anyone else, as in Mirrleesian models. In this case, (35) reads $\int_{y>t} \alpha dF = \Delta(t)$, so taxes are efficient if and only if $\Delta$ is nonincreasing and bounded between zero and one, as in Bierbrauer et al. (2023). With complementarities, the test can be implemented as in the family-based case, by using a linear programming solver to check whether (35) admits a solution with nonnegative Pareto weights.

## 5.3 Complementarities and taxation in Beckerian economies

In our economy, sorting and consumption sharing occur both because of complementarities in labor supply and because of non-pecuniary preference shocks. As we discussed in footnote 2, our formulation of non-pecuniary shocks is also isomorphic to adding an arbitrary symmetric component $\Phi(w,w')$ to family payoffs that is independent of consumption and labor supply. Although complementarities in labor supply and non-pecuniary complementarities can have similar qualitative effects for sorting and consumption sharing, they have different implications for taxation. Complementarities in labor supply give rise to the sorting, sharing, and indirect behavioral effects of taxation, whereas these three effects are absent if sorting is driven purely by non-pecuniary motives.

Consider an economy in which there are no complementarities and taxation is individual-based. In such an economy, individuals may have non-trivial sorting and sharing patterns for non-pecuniary reasons. However, as the next proposition shows, the effects of taxation in such economies are the same as in the traditional Mirrleesian analysis. In particular, optimal taxation in such models coincides with that of Diamond (1998), who studied optimal taxation in the Mirrleesian model with quasi-linear preferences.

**Proposition 5.** *Suppose there are no complementarities, so that $\mathcal{N}$ takes the form $\mathcal{N}(l,l') = -N(l) - N(l')$ for some function $N$. Then the optimal taxes on individual earnings coincide with the optimal taxes in the Mirrleesian economy with the same primitives $N$, $G$ and $\alpha$. In particular, the optimal tax formula is the same as in Diamond (1998) and can be written as*

$$\frac{\partial T(t)/\partial y}{1-\partial T(t)/\partial y} = \frac{1}{\bar{\xi}(t)}\frac{1-H(t)}{h(t)t}\mathbb{E}\left[1-\alpha|y>t\right].$$

Proposition 5 motivates the following decomposition of (36) into Mirrleesian and Beckerian effects.

$$\frac{\partial T(t)/\partial y}{1-\partial T(t)/\partial y} = \underbrace{-\mathbb{E}\left[\frac{\zeta}{\bar{\xi}(t)}\frac{\partial T^{\mathfrak{s}}/\partial y}{1-\partial T^{\mathfrak{s}}/\partial y}|y=t\right]}_{\text{indirect behavioral}} + \underbrace{\frac{1}{\bar{\xi}(t)}\times\frac{1-H(t)}{h(t)t}\times\Big\{\mathbb{E}[1-\alpha|y>t]}_{\text{Mirrleesian}} \tag{37}$$

$$+\underbrace{\mathbb{E}\left[\alpha-\mathbf{K}^*\alpha-(\mathbf{K}^*\alpha)^{\mathfrak{s}}\,|y>t\right]}_{\text{sharing}}\underbrace{-\mathbb{E}\left[\frac{1}{2}\mathbf{M}^*(T+T^{\mathfrak{s}})+\frac{1}{2}(\mathbf{M}^*(T+T^{\mathfrak{s}}))^{\mathfrak{s}}|y>t\right]}_{\text{sorting}}\Big\}.$$

This formula isolates the contribution of each of the three Beckerian mechanisms to optimal taxes. We quantify them in the next section.

# 6 Quantitative illustration

The optimal tax formulas derived in Propositions 1 and 3 not only provide qualitative insights about the economic forces that shape optimal taxes but also offer a way to compute those taxes numerically. The elasticities $\xi$ and $\zeta$, the earnings distribution $H$, and the operators $\mathbf{K}^*$ and $\mathbf{M}^*$ are endogenous objects that depend implicitly on taxes. One can treat equations (33) and (36) as fixed point problems that determine optimal family- and individual-based taxes. In this section, we describe how these equations can be solved and discuss their quantitative implications in a calibrated economy. We focus on individual-based taxation because it allows a particularly transparent separation of Mirrleesian and Beckerian effects using decomposition (37).

We assume that the function $\mathcal{N}$ takes the form

$$\mathcal{N}(l,l') = -\left(\frac{1}{1+1/\sigma}(l)^{(1+\varrho)(1+1/\sigma)} + \frac{1}{1+1/\sigma}(l')^{(1+\varrho)(1+1/\sigma)}\right)^{1/(1+\varrho)}, \tag{38}$$

that the distribution of productivities $G$ is Pareto-lognormal, and that non-pecuniary shocks follow the logit model. We make these choices for transparency. In (38), the parameter $\varrho$ captures complementarities. If $\varrho = 0$, then $\mathcal{N}(l,l') = -\frac{1}{1+1/\sigma}(l)^{1+1/\sigma} - \frac{1}{1+1/\sigma}(l')^{1+1/\sigma}$ is a standard isoelastic function with elasticity $\sigma$, and it is used extensively in applied work. Complementarities are positive if $\varrho > 0$ and negative if $\varrho < 0$. The Pareto-lognormal distribution is the standard choice in the quantitative tax literature because it captures well the cross-sectional distribution of individual earnings. The logit model is used extensively in the family economics literature (Choo and Siow (2006), Gayle and Shephard (2019)), and we show in this section how numerical analysis can be simplified for this distribution.

## 6.1 Calibration

To calibrate our economy, we use data from the 2021 March Supplement of the Current Population Survey (CPS). We restrict attention to households where both spouses are present, are between the ages of 25 and 65, worked at least 20 weeks in the previous year, and are not self-employed or family workers. Following Golosov and Krasikov (2025), we symmetrize the dataset by creating one more copy of each couple with permuted earnings. We assume that all earnings are taxed at a constant marginal tax rate $\tau$, which we set to 30 percent.

Under this linear tax, the function (38) admits a simple relationship between the parameters $(\sigma, \varrho)$ and the elasticities $(\xi, \zeta)$, which is given by

$$\xi = \frac{1}{(1+\varrho)(1+1/\sigma)-1} + \frac{\varrho(\sigma+1)}{(1+\varrho)(1+1/\sigma)-1}\mathrm{s}, \quad \zeta = \frac{\varrho(\sigma+1)}{(1+\varrho)(1+1/\sigma)-1}(1-\mathrm{s}), \tag{39}$$

where s equals the individual's share of family earnings, i.e., $\mathrm{s} := y/Y$. Since the population average of s must be equal to $\frac{1}{2}$, equation (39) gives a one-to-one relationship between the parameters $(\sigma, \varrho)$ and the average elasticities $(\mathbb{E}\xi, \mathbb{E}\zeta)$. In our calibration, we target an average own elasticity of labor supply of $\mathbb{E}\xi = \frac{1}{3}$, which lies in the middle of the range considered by Diamond (1998). For the cross-elasticity, we rely on the results of Calvo et al. (2024), who estimate complementarities in home production using time use data from the German Socio-Economic Panel (GSOEP). They find that these complementarities are positive across various socio-economic groups. Converted into elasticities, their estimates imply that the cross-elasticity is between one quarter and one half of the own elasticity of labor supply (see appendix for details). Given these estimates, we choose a conservative target for the average cross-elasticity, $\mathbb{E}\zeta = \frac{1}{4}\mathbb{E}\xi = \frac{1}{12}$. These targets yield

$$\sigma = \mathbb{E}\xi + \mathbb{E}\zeta \approx 0.42, \quad \varrho = \frac{1+\frac{1}{\mathbb{E}\xi-\mathbb{E}\zeta}}{1+\frac{1}{\mathbb{E}\xi+\mathbb{E}\zeta}} - 1 \approx 0.47.$$

To calibrate the distribution of productivities $G$, we invert the optimality conditions of households. Under the linear tax, a couple with earnings $(y, y')$ has productivities

$$w = \frac{(y)^{\frac{1/\sigma}{1+1/\sigma}}}{(1-\tau)^{\frac{1}{1+1/\sigma}}}\left[(1+1/\sigma)\mathrm{s}\right]^{\frac{\varrho}{(1+\varrho)(1+1/\sigma)}}, \quad w' = \frac{(y')^{\frac{1/\sigma}{1+1/\sigma}}}{(1-\tau)^{\frac{1}{1+1/\sigma}}}\left[(1+1/\sigma)(1-\mathrm{s})\right]^{\frac{\varrho}{(1+\varrho)(1+1/\sigma)}}.$$

Given the observed earnings $(y, y')$ and calibrated $(\sigma, \varrho)$, we recover the productivity for each person in the sample and, hence, the empirical productivity distribution. We smooth it by fitting a Pareto-lognormal (PLN) distribution that matches the mean, the Gini coefficient, and the Pareto tail statistics of the empirical marginal distribution of productivities. All three targets have simple analytical expressions in terms of the three parameters of the PLN distribution. We provide additional details and show the goodness-of-fit in the appendix.

Given the complementarities, the parameter $\beta$ controls the assortativeness of matches. We set $\beta$ so that the calibrated model matches the rank correlation (Kendall's tau) between spouses' productivities that we recover from the data. This correlation is 0.1 in our sample, which yields $\beta = 33$.

## 6.2 Numerical implementation

We find optimal taxes by iterating on equation (36). We start with a guess of the optimal tax $T$, which determines the left-hand side of (36). Given this $T$, we find the family surplus $v$, the equilibrium $(u, q)$, the matching elasticities $\gamma$, and the operators $\mathbf{K}$ and $\mathbf{M}$. We then construct the right-hand side of equation (36). If the difference between the two sides of equation (36) exceeds a tolerance bound, we update our guess for $T$ and repeat the procedure until the fixed point of (36) is found.

To implement this approach numerically, we fix an equally log-spaced grid of earnings $\{y_i\}_{i=1}^{N_y}$. We parametrize the candidate tax function by its marginal tax rates $\{\tau_i\}_{i=1}^{N_y}$ at the grid points. We use cubic splines to interpolate these marginal tax rates between the grid points and integrate them to construct (up to a constant that is irrelevant for our analysis) the tax function $T$. We define the equally log-spaced productivity grid $\{w_i\}_{i=1}^{N_w}$, whose first and last points are chosen so that the earnings of each spouse in the couples $(w_1, w_1)$ and $(w_{N_w}, w_{N_w})$ are given by $y_1$ and $y_{N_y}$, respectively. We then compute the optimal earnings $\{(y_{w_i}(w_j), y_{w_j}(w_i))\}_{i,j=1}^{N_w}$ and the family surpluses $\{v(w_i, w_j)\}_{i,j=1}^{N_w}$ for all pairs of grid productivities.

To find the equilibrium, we use the fact that in the logit model, the equilibrium conditions imply

$$u_w(w') = \frac{v(w,w') + U_w(u_w) - U_{w'}(u_{w'})}{2}, \quad q_w(w') = \exp\left(\beta \frac{v(w,w') - U_w(u_w) - U_{w'}(u_{w'})}{2}\right) g(w'), \tag{40}$$

where the expected utilities $(U_w(u_w))_w$ solve

$$U_w(u_w) = \frac{2}{\beta} \ln\left(\sum_{w'} \exp\left(\beta \frac{v(w,w') - U_{w'}(u_{w'})}{2}\right) g(w')\right). \tag{41}$$

We first use (41) to find values of expected utilities $\{U_{w_i}(u_{w_i})\}_{i=1}^{N_w}$ and then use (40) to construct values of $(u, q)$ as well as the matrix of matching elasticities $\gamma$ on our productivity grid.[20]

We compute the sorting and consumption sharing effects using two different methods. In the first method, we follow the approach that we described in Section 4 to construct the matrices $\mathbf{K}$ and $\mathbf{M}$. This approach is general, as it applies to any distribution of non-pecuniary shocks, but requires the inversion of the matrix $\mathbf{I} - \mathbf{A} + \mathbf{B}$. This matrix $\mathbf{I} - \mathbf{A} + \mathbf{B}$ has dimension $N_w^2 \times N_w^2$, so its inversion can be time consuming when the grid of productivities is large.

[20] In our computations, we allow for very large values of $w$. This makes it preferable to solve equation (41) in two steps. We first introduce the change of variables $\tilde{U}_w := U_w(u_w) - \frac{1}{2}v(w,w)$ and use Newton iterations to solve (41) for $\{\tilde{U}_{w_i}\}_{i=1}^{N_w}$. Then, we recover $\{U_{w_i}(u_{w_i})\}_{i=1}^{N_w}$ from $\{\tilde{U}_{w_i}\}_{i=1}^{N_w}$. Using (41) directly to find $\{U_{w_i}(u_{w_i})\}_{i=1}^{N_w}$ causes numerical overflow, because $\exp(v(w,w))$ increases fast in $w$.

An alternative method exploits the specific structure of logit shocks and is computationally faster. With logit shocks, one can differentiate equation (41) to obtain

$$\dot{U}_w + \sum_{w'} q_w(w')\dot{U}_{w'} = \sum_{w'} q_w(w')\dot{v}(w,w'). \tag{42}$$

We solve this equation for $\dot{U}_w$ and then use (40) to find the responses $(\dot{u}, \dot{q})$ to perturbations $\dot{v}$ as follows. Write the left-hand side of equation (42) as $(\mathbf{I}+\mathbf{Q})\dot{U}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{Q}$ consists of $\{q_w(w')\}_{w,w'}$, and let $\mathbf{P} := (\mathbf{I}+\mathbf{Q})^{-1}$. The solution to (42) can be written as

$$\dot{U}_{\tilde{w}} = \sum_{w,w'} p_{\tilde{w}}(w) q_w(w') \dot{v}(w,w'),$$

where $\{p_w(w')\}_{w,w'}$ are the elements of $\mathbf{P}$. Differentiating (40) and substituting the solution $\dot{U}_w$, we can express $\dot{u}$ and $\dot{q}$ as

$$\dot{u}_{\tilde{w}}(\tilde{w}') = \sum_{w,w'} \hat{\mathbf{K}}_{\tilde{w},\tilde{w}'}(w,w')\dot{v}(w,w'), \quad \frac{\dot{f}(\tilde{w},\tilde{w}')}{f(\tilde{w},\tilde{w}')} = \sum_{w,w'} \hat{\mathbf{M}}_{\tilde{w},\tilde{w}'}(w,w')\dot{v}(w,w'),$$

where the matrices $\hat{\mathbf{K}}$ and $\hat{\mathbf{M}}$ are given by

$$\begin{aligned}
\hat{\mathbf{K}}_{\tilde{w},\tilde{w}'}(w,w') &= \frac{1}{2} \times \left\{ 1_{w=\tilde{w}} 1_{w'=\tilde{w}'} + p_{\tilde{w}}(w) q_w(w') - p_{\tilde{w}'}(w) q_w(w') \right\}, \\
\hat{\mathbf{M}}_{\tilde{w},\tilde{w}'}(w,w') &= \frac{\beta}{2} \times \left\{ 1_{w=\tilde{w}} 1_{w'=\tilde{w}'} - p_{\tilde{w}}(w) q_w(w') - p_{\tilde{w}'}(w) q_w(w') \right\}.
\end{aligned}$$

The matrices $\hat{\mathbf{K}}$, $\hat{\mathbf{M}}$ are not the same as $\mathbf{K}$, $\mathbf{M}$, but they satisfy $\mathbf{K}\dot{\mathcal{T}} = \hat{\mathbf{K}}\dot{\mathcal{T}}$ and $\mathbf{M}\dot{\mathcal{T}} = \hat{\mathbf{M}}\dot{\mathcal{T}}$ for symmetric tax perturbations $\dot{\mathcal{T}}$. Since tax perturbations must be symmetric, we can use $(\hat{\mathbf{K}}, \hat{\mathbf{M}})$ instead of $(\mathbf{K}, \mathbf{M})$ in all our constructions. The main computational advantage of this alternative method is that it only requires the inversion of the matrix $\mathbf{I}+\mathbf{Q}$, which has dimension $N_w \times N_w$.[21]

We tested both methods of computing the sorting and sharing effects. Both methods produce the same results, but the second one is faster. With $N_w = 200$ grid points for productivities, constructing these effects takes 577 seconds with the first method and 32 seconds with the second one on a MacBook laptop, using standard Python matrix inversion routines.

Once we have computed the behavioral, sorting and sharing responses in the space of productivities, we convert them into the space of earnings by building the conditional expectations that appear in (36). We leave the details of its implementation for the appendix.

[21] As discussed in Section 4, the spectral radius of $\mathbf{A}-\mathbf{B}$ is less than one, which allows us to write the inverse as a geometric series $(\mathbf{I}-\mathbf{A}+\mathbf{B})^{-1} = \sum_{k=0}^{\infty}(\mathbf{A}-\mathbf{B})^k$. In contrast, $\mathbf{Q}$ is a stochastic matrix, so its spectral radius equals one. This implies that the matrix $\mathbf{P} = (\mathbf{I}+\mathbf{Q})^{-1}$ cannot be written as the standard Neumann series. Instead, $\mathbf{P}$ has a representation $\mathbf{P} = \sum_{k=0}^{\infty}(-1)^k\left(\mathbf{Q}^k - \mathbf{1}g^{\mathrm{T}}\right) + \frac{1}{2}\mathbf{1}g^{\mathrm{T}}$, where $\mathbf{1}$ is the vector of ones. Since $g^{\mathrm{T}}\mathbf{Q} = g^{\mathrm{T}}$, the matrix $\mathbf{1}g^{\mathrm{T}}$ is the projection onto the eigenvector of $\mathbf{Q}$ with the unit eigenvalue. The subtraction of this term removes the unit-eigenvalue component of $\mathbf{Q}^k$ ensuring convergence of the infinite sum, and the addition of $\frac{1}{2}\mathbf{1}g^{\mathrm{T}}$ accounts for the action on this invariant subspace.

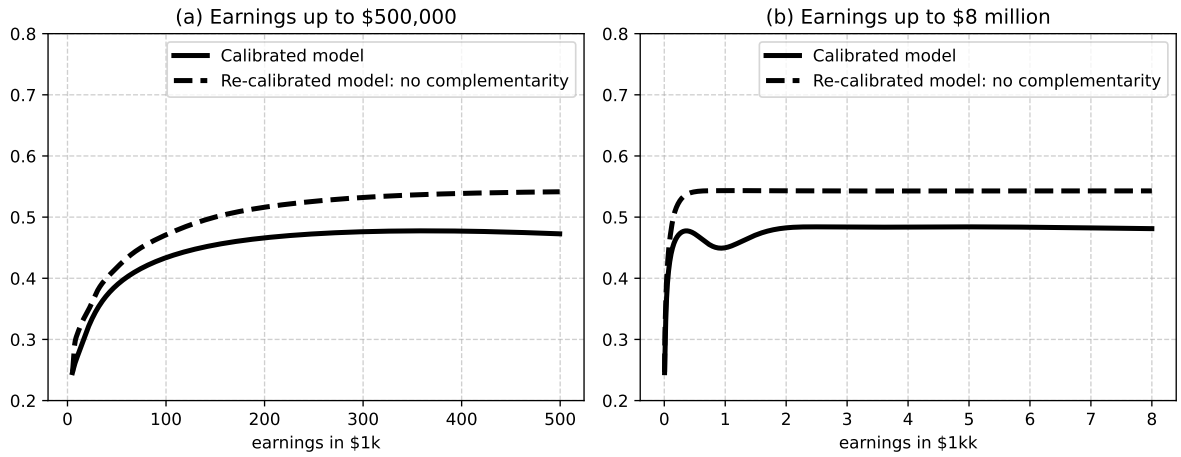



Figure 2: Optimal tax rates $\partial T/\partial y$. Solid line: calibrated model. Dashed line: recalibrated model without complementarities ($\varrho = 0$).

## 6.3 Quantitative results

We plot the optimal marginal tax rates for Pareto weights $\alpha(w) \propto \exp(-w)$ as a solid line in Figure 2. To isolate the role of complementarities, we compare them to the optimal marginal tax rates in a model without complementarities that is otherwise calibrated to the same moments. In particular, we set $\varrho = 0$ and $\sigma = \frac{1}{3}$, so that the own labor supply elasticity $\xi$ is still $\frac{1}{3}$ (the cross-elasticity $\zeta$ is naturally zero), and recalibrate $G$ to match the same distribution of individual earnings under the stylized U.S. tax schedule.[22] We plot the resulting optimal tax rates as a dashed line in Figure 2.

Panel (a) of Figure 2 shows that the two schedules nearly coincide at the very bottom of the earnings distribution, where marginal tax rates are about 25 percent, and diverge as earnings rise, reaching a difference of about 7 percentage points at the earnings level of $500,000, which covers all but about 0.3 percent of individuals in our economy. Panel (b) extends the earnings grid further up to 8 million dollars and shows that both schedules level off and the difference between them remains even at the very top. Thus, optimal marginal taxes are lower for most earners and less progressive once complementarities are taken into account. The differences are sizable: marginal tax rates are about 3 to 4 percentage points lower between $50,000 and $100,000, 7 percentage points lower at $500,000, and 6 to 9 percentage points lower above that. Panel (b) also shows that, unlike the schedule without complementarities, which increases monotonically, the schedule with complementarities is non-monotone: it peaks at about 48 percent between $300,000 and $400,000, dips to about 45 percent around $1 million, and then returns to about 48 percent.

[22] In the absence of complementarities, matching in the logit model is random, so this model cannot match the correlation of spousal productivities in the data. While this correlation can be matched with other specifications of $\Psi_w$, such as the FC-MNL specification used by Galichon and Salanié (2022), we know from Proposition 5 that, in the absence of complementarities, optimal taxes are the same for all $\Psi_w$.

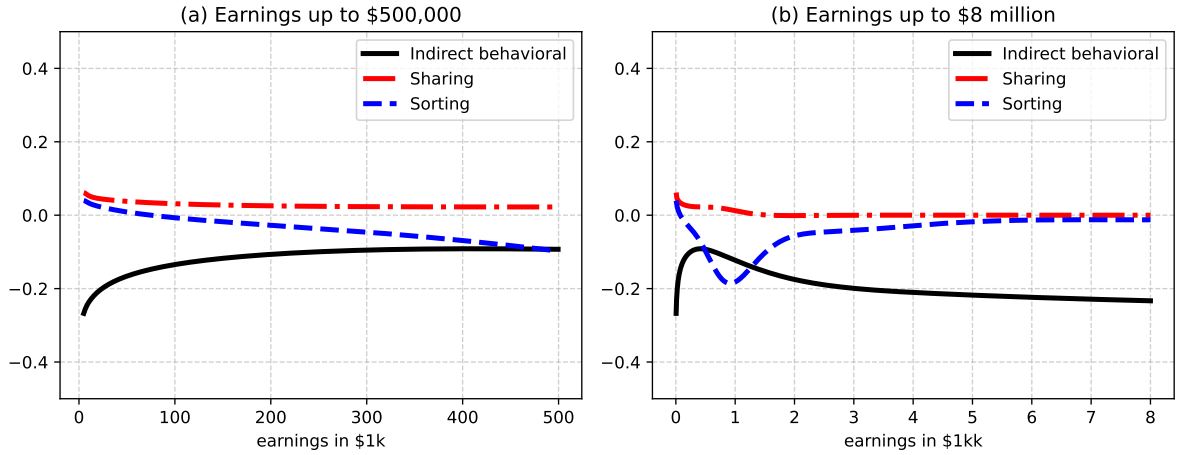



Figure 3: The three Beckerian terms in decomposition (37).

To understand what drives these differences, we plot separately the three Beckerian effects from the decomposition (37) in Figure 3.[23] Panel (a) shows that the indirect behavioral effect is negative, so that it lowers optimal marginal taxes, the sharing effect is positive, and the sorting effect is positive for low earners and negative for high earners. Panel (b) shows that the sorting effect is U-shaped, with its minimum at about $900,000, and that both the sorting and sharing effects converge to zero at the very top, while the indirect behavioral effect remains sizable. We now explain the economic intuition behind these results.

We start with the indirect behavioral effect. Under specification (38), the own behavioral elasticity $\bar{\xi}(t)$ is roughly constant across earnings levels, but the ratio $\mathbb{E}[\zeta|y=t]/\bar{\xi}(t)$ is not: by (39), it decreases with the individual's share of family earnings. In our calibration, this ratio is about $\frac{1}{4}$ around median earnings of $52,000 and at the very top, where spouses earn about the same, larger at the bottom, where individuals are typically secondary earners, and smaller between $200,000 and $500,000, where they are typically primary earners. Thus, the indirect behavioral effect lowers $\frac{\partial T(t)/\partial y}{1-\partial T(t)/\partial y}$ by about 20 percent around median earnings and at the top, by more at the bottom, and by less in between.

The sorting and sharing effects can be understood from our two-type example in Section 3. In that example, we showed, using the two tax reforms depicted in Figure 1, that the sorting effect increases the cost of raising marginal tax rates at high earnings levels but decreases it at low earnings levels. Consistent with our discussion in that section, Figure 3 shows that the sorting effect (dashed line) is positive at low earnings and becomes negative at approximately $74,000. The sorting effect is most negative at about $900,000, which explains the dip in the optimal marginal tax rate around $1 million

[23] In addition to the three Beckerian effects, the solid and dashed lines in Figure 2 differ because the Mirrleesian term $\frac{1}{\bar{\xi}(t)}\frac{1-H(t)}{h(t)t}\mathbb{E}[1-\alpha|y>t]$ differs across the two models. This term is endogenous and depends on the underlying tax schedule. Numerically, the difference between the values of this term in the two models is small and does not affect our discussion below.

in panel (b) of Figure 2. We also showed in the two-type economy that both reforms in Figure 1 increase redistribution from high- to low-productivity spouses through endogenous consumption sharing. Our social planner values this redistribution, and the sharing effect (dashed-dotted line) in Figure 3 is positive. Breaking up assortative matches among high-productivity individuals leads to large losses of surplus, so the sorting effect dominates the sharing effect for high earners, while the sharing effect dominates for low earners. Finally, panel (b) of Figure 3 shows that both effects converge to zero at the very top. For very productive individuals, the pecuniary gains from marrying a similarly productive spouse are large relative to non-pecuniary motives, so matching is nearly assortative and responds little to taxes.

# 7 Taxation in other matching environments

In our baseline economy, agents are identical in all dimensions other than labor market productivity and have quasi-linear preferences, matching is bilateral, and surplus sharing is competitive. These features allowed us to streamline the exposition. The general approach that we developed in the previous sections to study the effects of taxation does not require these features and extends to a wide variety of endogenous matching environments. We discuss these extensions in this section. The appendix contains additional details and proofs.

## 7.1 Heterogeneity, asymmetries, and outside options

Suppose that heterogeneity is described by a vector of characteristics $w$ drawn from a distribution $G$. We write the family surplus in terms of the optimal choice of consumption and labor earnings as

$$v(w,w') := \max_{(c,c',y,y')} c + c' + \mathcal{N}(y,y';w,w') \tag{43}$$

subject to

$$c + c' \leq y + y' - \mathcal{T}(y,y'),$$

where $\mathcal{T}$ is symmetric in $y$ and $y'$, and $\mathcal{N}$ is symmetric in $(y,w)$ and $(y',w')$. Given this $v$, we define the matching equilibrium as in Section 2.3. It is easy to verify that all key results of our analysis, such as Lemmas 1 and 3, and Propositions 1, 2, 3 and 4 hold as stated, since none of them exploits the structure of $\mathcal{N}$ or the dimensionality of $w$. Moreover, a version of Proposition 5, which showed that the Beckerian effects are closely linked to complementarities in labor supply, also holds in this economy.

Since the function $\mathcal{N}$ may depend on $w$ in an arbitrary way, the distribution $G$ allows for any correlation of individual attributes, and the law of non-pecuniary shocks $\Psi_w$ may have any dependence of shocks $\epsilon(w')$ on $w$, this specification is very rich. It includes environments in which individuals differ in market and home productivity, and have heterogeneous preferences over home production, market

work, and marriage partners. For example, by designating one of the dimensions of $w$ as a binary sex marker, $s \in \{m, f\}$, and having other elements of $w$ be dependent on $s$, this environment includes settings in which $m$ and $f$ types have different cross-sectional distributions of productivities, different comparative advantages in home or market work, and strong non-pecuniary preferences for forming $mf$ matches.

This specification also clarifies the role of the symmetry assumption in our tax analysis. The key economic content of this assumption is that the tax function treats the two spouses symmetrically, e.g., a couple in which the $m$ spouse earns $y$ and the $f$ spouse earns $y'$ faces the same tax as a couple in which the $m$ spouse earns $y'$ and the $f$ spouse earns $y$. For such symmetric tax functions, it is without loss of generality to model the surplus function $v$ and the marriage market in a symmetric fashion as well. On the other hand, if the tax system treats the two spouses differently, for example, by imposing different taxes on $m$ and $f$ types, it may be more convenient to formulate the equilibrium as a bilateral matching between $m$ and $f$.[24] The analysis of such economies is often simpler, because reforms of sex-based taxes affect only one side of the market at a time. We illustrate this in the appendix by first considering an arbitrary bilateral matching economy, in which the two sides of the market may face different taxes, and then introducing a restriction that taxes must be symmetric, which reduces it to our baseline settings. We also show in the appendix that it is straightforward to add the option of remaining single, and to allow taxes that treat single and married households differently.

## 7.2 Taxation with imperfectly transferable utility

In our baseline environment, we assume that preferences are quasi-linear in consumption. In the matching literature, such a specification of preferences is referred to as perfectly transferable utility, and it is known to simplify the analysis. In this section, we extend our approach to imperfectly transferable utility. To keep our discussion as close as possible to the baseline economy, we assume that each person has preferences $V\left(c - N(l, l')\right)$ over own consumption $c$, own labor $l$, and partner's labor $l'$, where $V$ is a strictly increasing and concave utility transformation. As in Section 2, we assume that spouses choose consumption and earnings efficiently.

When $V$ is linear, this environment collapses to our baseline model, and any point on the intra-family Pareto frontier maximizes the sum of spouses' utilities. When $V$ is non-linear, the Pareto frontier can be found by solving

$$\max_{(c,c',y,y')} \alpha V\left(c - N(y/w, y'/w')\right) + \alpha' V\left(c' - N(y'/w', y/w)\right) \text{ s.t. } c + c' = y + y' - \mathcal{T}(y, y'),$$

[24] In family economics, the bilateral formulation is somewhat restrictive, since it ignores the possibility of $mm$ and $ff$ matches, which our formulation handles easily. In other environments, such as matching of workers and firms, the bilateral formulation is more natural. We illustrate this in Section 7.3.

where $\alpha$ and $\alpha'$ are intra-household Pareto weights. This problem can equivalently be written in utils as

$$\max_{(u,u',y,y')} \alpha u + \alpha' u' \text{ s.t. } V^{-1}(u) + V^{-1}(u') = y + y' - \mathcal{T}(y,y') + \mathcal{N}\left(\frac{y}{w}, \frac{y'}{w'}\right),$$

where $\mathcal{N}(l,l') := -N(l,l') - N(l',l)$. This formulation suggests the following two-step procedure for characterizing efficient allocations. First, find the family surplus $v$ and the optimal earnings by solving

$$v(w,w') := \max_{(y,y')} y + y' - \mathcal{T}(y,y') + \mathcal{N}\left(\frac{y}{w}, \frac{y'}{w'}\right),$$

which is exactly the same problem as the one that defines the family surplus in equation (5) in our baseline economy. Second, find the spousal utilities by maximizing $\alpha u + \alpha' u'$ subject to $V^{-1}(u) + V^{-1}(u') = v(w,w')$. Since $\alpha, \alpha'$ are parameters here, we capture the whole intra-household Pareto frontier by varying them. Equivalently, we can simply trace the Pareto frontier by varying $u'$ and obtaining $u$ as a function of it. Given this observation, we can define matching equilibrium analogously to our definition in Section 2.3 except that equation (8) in Condition (C) is replaced by

$$V^{-1}(u_w(w')) + V^{-1}(u_{w'}(w)) = v(w,w') \quad \forall w, w'. \tag{44}$$

The mechanical and behavioral effects of taxation with imperfectly transferable utility are the same as in Section 4, but the sorting and sharing effects are slightly different. Recall that in the baseline economy, the sorting and sharing effects are characterized by the matching elasticities $\gamma$. When utility is perfectly transferable, $\gamma$ captures the responses of choice probabilities both to changes in utilities, i.e., $\gamma_w(\tilde{w}'|w') = -\partial \ln q_w(w')/\partial u_w(\tilde{w}')$, and to changes in consumption, $\gamma_w(\tilde{w}'|w') = -\partial \ln q_w(w')/\partial c_w(\tilde{w}')$. This equivalence breaks down when preferences are not quasi-linear, and our analysis requires some modifications.

Let $\gamma$ be the matching elasticities defined in (14). They are the semi-elasticities of choice probabilities with respect to utils, $\gamma_w(\tilde{w}'|w') = -\partial \ln q_w(w')/\partial u_w(\tilde{w}')$, and, as in the baseline economy, they are fully determined by the law of non-pecuniary shocks $\Psi_w$. Let $\frac{\partial V_w(w')}{\partial c} := V'\left(V^{-1}(u_w(w'))\right)$ denote the marginal utility of consumption of type $w$ in a match with $w'$, and define $\gamma^c$ as the marginal utility adjusted analogue of $\gamma$, i.e., $\gamma^c_w(\tilde{w}'|w') := \frac{\partial V_w(w')}{\partial c}\gamma_w(\tilde{w}'|w')$. Similarly, let $\bar{\gamma}^c_w(w') := \frac{\partial V_w(w')}{\partial c}\bar{\gamma}_w(w')$. By the chain rule, $\bar{\gamma}^c_w(w') = \partial \ln q_w(w')/\partial c_w(w')$, i.e., it is the semi-elasticity of $q_w(w')$ with respect to consumption in that match. Since $\bar{\gamma}^c_w(w') = \int \gamma^c_w(\tilde{w}'|w')d\tilde{w}'$, one can interpret $\gamma^c_w(\cdot|w')$ as a decomposition of this semi-elasticity into $\tilde{w}'$-specific components.

Consider a tax reform $\dot{\mathcal{T}}$. As in the baseline economy, $\dot{v} = -\dot{\mathcal{T}}$ by (13). Differentiating the equilibrium conditions, one finds that the utility response $\dot{u}$ satisfies the following integral equation:

$$\dot{u}_w(w') - \int \frac{\gamma^c_w(\tilde{w}'|w')}{\bar{\gamma}^c_w(w') + \bar{\gamma}^c_{w'}(w)} \dot{u}_w(\tilde{w}') d\tilde{w}' + \frac{\partial V_w(w')/\partial c}{\partial V_{w'}(w)/\partial c} \int \frac{\gamma^c_{w'}(\tilde{w}|w)}{\bar{\gamma}^c_w(w') + \bar{\gamma}^c_{w'}(w)} \dot{u}_{w'}(\tilde{w}) d\tilde{w}$$
$$= \frac{\bar{\gamma}^c_{w'}(w)}{\bar{\gamma}^c_w(w') + \bar{\gamma}^c_{w'}(w)} \frac{\partial V_w(w')}{\partial c} \dot{v}(w, w').$$

This equation can be written in operator notation as

$$\dot{u} = \mathbf{\Theta}^c \dot{v} + \mathbf{A}^c \dot{u} - \mathbf{B}^c \dot{u}, \tag{45}$$

where $\mathbf{\Theta}^c$, $\mathbf{A}^c$, $\mathbf{B}^c$ are analogous to $\mathbf{\Theta}$, $\mathbf{A}$, $\mathbf{B}$ in our baseline economy. We summarize the sorting and sharing effects with imperfectly transferable utility in the following lemma.

**Lemma 4.** *The responses $\dot{u}$, $\dot{q}$ and $\dot{f}$ to a tax perturbation $\dot{\mathcal{T}}$ are given by $\dot{u} = -\mathbf{K}^c \dot{\mathcal{T}}$ and $\dot{q}/q = \dot{f}/f = -\mathbf{M}^c \dot{\mathcal{T}}$, where $\mathbf{K}^c = (\mathbf{I} - \mathbf{A}^c + \mathbf{B}^c)^{-1} \mathbf{\Theta}^c$ and $\mathbf{M}^c = \mathbf{C}\mathbf{K}^c$, and $\mathbf{C}$ is as defined in Lemma 1.*

The rest of the analysis of taxation is very similar to that in the baseline economy. The formulas for the revenues and welfare responses in equations (25) and (28), the optimal tax formulas in Propositions 1 and 3, and the efficiency tests in Propositions 2 and 4 are the same as in the baseline economy, except that the operators $\mathbf{K}$ and $\mathbf{M}$ are replaced with their marginal utility adjusted analogues $\mathbf{K}^c$ and $\mathbf{M}^c$, and the Pareto weights are normalized so that the Lagrange multiplier on the budget constraint equals one. The main substantive difference is that tax reforms may have sorting and sharing effects even in the absence of labor complementarities.

Lemma 4 highlights that the matching elasticities $\gamma$ and their marginal utility adjusted analogue $\gamma^c$ play distinct roles in tax analysis. The effect of tax perturbations on agents' utilities is captured by $\mathbf{K}^c$, which is constructed from $\gamma^c$ and the marginal utilities. The relationship between sorting responses and changes in utilities $\dot{u}$ is given by $\dot{q}/q = \dot{f}/f = \mathbf{C}\dot{u}$, where the operator $\mathbf{C}$ is constructed solely from $\gamma$. One can use this observation to separately identify local properties of $\Psi$ and $V$ in empirical work. To do so, it is more convenient to define equilibrium not in terms of utilities $u$, which are unobservable to an econometrician, but in terms of consumption $c$, which can be measured in the data. One can then define matching elasticities in terms of changes in consumption rather than utils, i.e.,

$$\tilde{\gamma}_w(\tilde{w}'|w') = -\partial \ln q_w(w')/\partial c_w(\tilde{w}') = \frac{\partial V_w(\tilde{w}')}{\partial c} \gamma_w(\tilde{w}'|w').$$

Consumption responses $\dot{c}$ satisfy $\dot{c} = -\tilde{\mathbf{K}}\dot{\mathcal{T}}$, where $\tilde{\mathbf{K}}$ is constructed analogously to $\mathbf{K}$ by replacing $\gamma$ with $\tilde{\gamma}$. Elasticities $\tilde{\gamma}$ can be either estimated directly, for example by instrumenting for consumption changes, or inferred from observed responses to tax reforms. The relationship between sorting and consumption responses can be written as $\dot{f}/f = \tilde{\mathbf{C}}\dot{c}$, where $\tilde{\mathbf{C}}$ is constructed from $\tilde{\gamma}$ adjusted by the inverse of the marginal utility of consumption. Since $\dot{f}/f$, $\dot{c}$ and $\tilde{\gamma}$ are observable, this allows one to back out $\frac{\partial V_w(w')}{\partial c}$ and, therefore, characterize local properties of $\Psi$.

### 7.3 Taxation and sorting in firms

Our approach to studying taxation extends beyond family economics. The same techniques can be used to study taxation in other matching environments, such as models of sorting between workers and firms, or between workers and cities. We illustrate such applications with a matching model of heterogeneous firms and workers in the spirit of Lamadon et al. (2022).

Let $w$ denote the type of a worker and $x$ denote the type of a firm. A worker of type $w$ employed by firm $x$ produces output $A(w,x)$. Each firm $x$ sets wages $y_x \equiv (y_x(w))_w$ for different types of workers and hires all workers willing to work at these wages. To focus on matching and sorting responses, we assume that each worker supplies one unit of labor inelastically so that their earnings equal their wages.[25] Workers face an income tax $T(y)$, so that their after-tax income is $y - T(y)$. As in the previous section, workers' utility of consumption is $V$, and we use $u_w(x) = V(y_x(w) - T(y_x(w)))$ to denote pecuniary utility of a worker of type $w$ employed by firm $x$.

In addition to differences in productivities, Lamadon et al. (2022) allow firms to differ in amenities, locations, and products, and they let workers choose firms based on wages as well as non-pecuniary preferences for amenities, locations, and products, and idiosyncratic preference shocks. Following our discussion in footnote 2, one can embed all these non-pecuniary differences in the law $\Psi_w$. The ex-ante utility of a worker of type $w$ can be written as

$$U_w(u_w) := \mathbb{E}_w \left[ \max_x u_w(x) + \frac{1}{\beta}\epsilon(x) \right], \tag{46}$$

which has the same form as $U_w$ in our baseline model, with firms $x$ in place of spouses $w'$. The derivative $q_w(x) = \partial_x U_w(u_w)$ is the choice probability. The joint density of firms and workers is $f(w,x) = q_w(x)g(w)$, and the profit of firm $x$ is

$$\pi_x := \int (A(w,x) - y_x(w)) q_w(x) g(w) dw. \tag{47}$$

This specification assumes that the demand for the good produced by firm $x$ is perfectly elastic, so that its price is independent of the output of firm $x$ and of any other firm $\tilde{x} \neq x$.

We first consider the competitive wage setting in which all firms pay workers their marginal products.

**Definition.** A competitive matching equilibrium given the tax $T$ consists of $(u, \pi, q, y)$ such that

(A). choice probabilities $q$ are optimal given $u$, i.e., $q_w(x) = \partial_x U_w(u_w)$ for all $(w,x)$;

(B). wages equal marginal products, $y_x(w) = A(w,x)$ for all $(w,x)$;

(C). the utilities of workers are $u_w(x) = V(y_x(w) - T(y_x(w)))$ and the profits of firms are given by (47).

[25] The extension of this model to elastic labor supply is straightforward, and behavioral effects mirror those in the baseline economy.

The competitive matching equilibrium in the firm–worker setting has many parallels with the equilibrium in our baseline economy. In both cases, Conditions (A)–(C) have a similar structure, with (A) describing optimal matching choices and (C) describing feasible ways to allocate surplus between agents. In the firm–worker setting, the surplus is shared through wages $y$, which are taxed directly. Although firm–worker matching is technically one-to-many, in which each firm matches with multiple workers, effectively only workers face a non-trivial choice problem, because firms employ all workers willing to work at the posted wage. Since we abstracted from labor supply, the surplus of worker $w$ and firm $x$ is fixed at $A(w,x)$ and independent of taxes.

Consider a tax reform $\dot{T}$. Since wages equal marginal products, Condition (B) implies that they do not respond to the reform, $\dot{y}=0$. Condition (C) then implies that

$$\dot{u}_w(x) = -\frac{\partial V_w(x)}{\partial c}\dot{T}(y_x(w)), \tag{48}$$

where $\frac{\partial V_w(x)}{\partial c}$ is the marginal utility of consumption, as in the previous section. Firms' profits remain zero, $\dot{\pi}=0$, and the sorting responses are obtained by differentiating (A),

$$\frac{\dot{f}(w,x)}{f(w,x)} = \frac{\dot{q}_w(x)}{q_w(x)} = \bar{\gamma}_w(x)\dot{u}_w(x) - \int \gamma_w(\tilde{x}|x)\dot{u}_w(\tilde{x})d\tilde{x}. \tag{49}$$

By substituting $\dot{u}$ from (48), one obtains the sorting response $\dot{f}/f$ as a function of tax changes. Using our convention that $\gamma^c_w(\tilde{x}|x) = \frac{\partial V_w(x)}{\partial c}\gamma_w(\tilde{x}|x)$, it can be written as $\dot{f}/f = -\mathbf{M}^c\dot{T}$, which has the same structure as those in Lemma 4. The effect of tax perturbations on revenues and welfare is $\dot{R} = \int \dot{T}dF - \int(\mathbf{M}^{c,*}T)\cdot\dot{T}dF$ and $\dot{W} = -\int \alpha^c\dot{T}dF$, where $\alpha^c$ is the marginal-utility adjusted Pareto weight and $\mathbf{M}^{c,*}$ is defined analogously to $\mathbf{M}^*$ in Section 5.

This model is particularly simple because we abstracted from any dependence of a firm's surplus on the choices of other firms. Consequently, marginal products are fixed, and one can find the sorting and sharing responses without solving integral equations such as (18). In richer environments, e.g., in which firms' prices depend on the output of other firms through general equilibrium interactions, such integral equations re-emerge (see also Sachs et al. (2020) for discussion of this point in a different context).

The analysis is similar if surplus sharing occurs non-competitively. Suppose that each firm $x$ sets wages $y \equiv (y_x(w))_w$ to maximize its profit (47). Since workers choose firms based on both pecuniary and non-pecuniary motives, each firm faces an upward-sloping labor supply curve. The wage optimality conditions can be written as

$$\underbrace{\frac{A(w,x)-y_x(w)}{y_x(w)}}_{:=m_x(w)} \times \underbrace{\frac{\partial \ln c_w(x)}{\partial \ln y_x(w)}}_{:=\lambda_w(x)} \times c_w(x)\overline{\gamma}^c_w(x) = 1, \tag{50}$$

where $c_w(x) := y_x(w) - T(y_x(w))$ is the after-tax wage. This equation is the familiar monopsonist's optimality condition. The variable $m_x(w)$ represents a mark-down of wages relative to the marginal

product. The term $\lambda_w(x) \times c_w(x)\bar{\gamma}_w^c(x)$ captures the elasticity of labor supply with respect to the wage paid by the firm. This elasticity is decomposed into $c_w(x)\bar{\gamma}_w^c(x)$, which is the elasticity of labor supply with respect to the after-tax wage, and $\lambda_w(x)$, which is the elasticity of the after-tax wage with respect to the pre-tax wage. The definition of the monopsonistic equilibrium is the same as the competitive equilibrium, except that condition (B) is replaced by (50).

To find the effects of taxes, we consider perturbations of equilibrium conditions. Equation (49) is unchanged, while the utility response of workers is

$$\dot{u}_w(x) = -\frac{\partial V_w(x)}{\partial c}\dot{T}(y_x(w)) + \frac{\partial V_w(x)}{\partial c} y_x(w) \left(1 - \frac{\partial}{\partial y} T(y_x(w))\right) \frac{\dot{y_x}(w)}{y_x(w)}. \tag{51}$$

Unlike in the competitive economy, wages respond to taxes under monopsony. Taking logs of (50) and differentiating, we obtain

$$\underbrace{-\left(1 + \frac{1}{m_x(w)}\right)\frac{\dot{y_x}(w)}{y_x(w)}}_{=\dot{m}_x(w)/m_x(w)} + \underbrace{\left(1 - \frac{y_x(w)\frac{\partial^2}{\partial y^2}T(y_x(w))}{1 - \frac{\partial}{\partial y}T(y_x(w))}\right)\frac{\dot{y_x}(w)}{y_x(w)} - \frac{1}{1 - \frac{\partial}{\partial y}T(y_x(w))}\frac{\partial}{\partial y}\dot{T}(y_x(w))}_{=\dot{\lambda}_w(x)/\lambda_w(x) + \dot{c}_w(x)/c_w(x)}$$
$$+ \underbrace{\frac{\dot{\bar{\gamma}}_w(x)}{\bar{\gamma}_w(x)} + \frac{\frac{\partial^2 V_w(x)}{\partial c^2}}{\left(\frac{\partial V_w(x)}{\partial c}\right)^2}\dot{u}_w(x)}_{=\dot{\bar{\gamma}}_w^c(x)/\bar{\gamma}_w^c(x)} = 0. \tag{52}$$

Suppose for a moment that $\bar{\gamma}$ is constant and does not respond to tax reforms. In this case, $\dot{\bar{\gamma}} = 0$ and one can find responses $(\dot{u}, \dot{y})$ by solving (51) and (52) as a function of $\dot{T}$ and $\frac{\partial}{\partial y}\dot{T}$. These responses can be found separately for each pair $(w, x)$ by solving a system of two equations in two unknowns. This solution then can be used in (49) to find $\dot{f}/f$. Unlike in our baseline economy, both the sorting and sharing responses depend not only on the changes in the levels of statutory taxes, $\dot{T}$, but also on statutory changes in marginal tax rates, $\frac{\partial}{\partial y}\dot{T}$. Consequently, two additional operators, similar to $\mathbf{K}$ and $\mathbf{M}$ in Lemma 1, describe the effects of changes in marginal tax rates $\frac{\partial}{\partial y}\dot{T}$ on equilibrium sorting and surplus sharing.

The own semi-elasticity $\bar{\gamma}$ is constant in the logit model, where it equals $\beta$ in all matches, as well as in all models that have the nested logit structure. More generally, $\bar{\gamma}$ may depend on the menu of pecuniary utilities available to workers. In such cases, one can no longer solve (51) and (52) separately for each pair $(w, x)$. Instead, the sharing responses $(\dot{u}, \dot{y})$ are characterized by an integral equation similar to (18) in our baseline economy. We illustrate this in the appendix using the flexible coefficient multinomial logit model (FC-MNL) that is used in industrial organization (Davis and Schiraldi (2014)) and family economics (Galichon and Salanié (2022)).

# 8 Conclusion

In this paper, we developed a general approach to studying the effects of taxation in environments with complementarities and endogenous matching. An important feature of such models is that tax changes affecting one group of agents can propagate through equilibrium adjustments in sorting and consumption sharing, thereby influencing outcomes for all agents in the economy. Using linear operator techniques, we characterized these adjustments for arbitrary tax reforms and, combining them with the labor supply responses, the effects of such reforms on tax revenues and social welfare. We used these results to derive formulas for optimal taxes and to characterize efficient taxes under family- and individual-based taxation. We also showed how these techniques can be used to compute optimal taxes numerically.

Our approach highlights that matching elasticities are among the key objects that determine the effects of taxation in such models. These elasticities are closely analogous to behavioral elasticities familiar from standard consumer theory. While there exists a vast literature that estimates behavioral elasticities, the body of work that estimates matching elasticities in Beckerian settings is much smaller. We view their systematic estimation as the natural next step in this research agenda.

# References


**Ales, Laurence, and Christopher Sleet.** 2016. "Taxing Top CEO Incomes." *American Economic Review*, 106(11): 3331–66.

**Ales, Laurence, and Christopher Sleet.** 2022. "Optimal Taxation of Income-Generating Choice." *Econometrica*, 90(5): 2397–2436.

**Becker, Gary S.** 1973. "A Theory of Marriage: Part I." *Journal of Political Economy*, 81(4): 813–846.

**Becko, John Sturm, and André Sztutman.** 2026. "Efficiency Criteria, Income Taxation, and Heterogeneous Elasticities." *American Economic Review*, 116(5): 1876–1913.

**Berger, David W., Kyle F. Herkenhoff, Jaehun Jeong, and Simon Mongey.** 2026. "Labor Market Power with Worker and Firm Heterogeneity." Working Paper 35640, National Bureau of Economic Research.

**Bergstrom, Katy, and William Dodds.** 2026. "A General Theory of Inverse Welfare Functions." *Theoretical Economics*, 21(3): 973–1018.

**Bierbrauer, Felix J., Pierre C. Boyer, and Emanuel Hansen.** 2023. "Pareto-Improving Tax Reforms and the Earned Income Tax Credit." *Econometrica*, 91(3): 1077–1103.

**Blundell, Richard, Mike Brewer, Peter Haan, and Andrew Shephard.** 2009. "Optimal Income Taxation of Lone Mothers: An Empirical Comparison of the UK and Germany." *Economic Journal*, 119(535): F101–F121.

**Bonhomme, Stéphane, Thibaut Lamadon, and Elena Manresa.** 2019. "A Distributional Framework for Matched Employer Employee Data." *Econometrica*, 87(3): 699–739.

**Calvo, Paula A., Ilse Lindenlaub, and Ana Reynoso.** 2024. "Marriage Market and Labour Market Sorting." *Review of Economic Studies*, 91(6): 3316–3361.

**Chiappori, Pierre-André, and Bernard Salanié.** 2023. "Mating Markets." In *Handbook of the Economics of the Family, Volume 1.* eds. by Shelly Lundberg, and Alessandra Voena, Amsterdam: Elsevier, 49–109.

**Chiappori, Pierre-André, Bernard Salanié, and Yoram Weiss.** 2017. "Partner Choice, Investment in Children, and the Marital College Premium." *American Economic Review*, 107(8): 2109–2167.

**Choo, Eugene, and Aloysius Siow.** 2006. "Who Marries Whom and Why." *Journal of Political Economy*, 114(1): 175–201.

**Colombi, Roberto.** 1990. "A New Model of Income Distribution: The Pareto-Lognormal Distribution." In *Income and Wealth Distribution, Inequality and Poverty*. eds. by Camilo Dagum, and Michele Zenga, Berlin: Springer, 18–32.

**Dagsvik, John K.** 1994. "Discrete and Continuous Choice, Max-Stable Processes, and Independence from Irrelevant Attributes." *Econometrica*, 62(5): 1179–1205.

**Dagsvik, John K.** 2000. "Aggregation in Matching Markets." *International Economic Review*, 41(1): 27–58.

**Davis, Peter, and Pasquale Schiraldi.** 2014. "The Flexible Coefficient Multinomial Logit (FC-MNL) Model of Demand for Differentiated Products." *RAND Journal of Economics*, 45(1): 32–63.

**Diamond, Peter A.** 1998. "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates." *American Economic Review*, 88(1): 83–95.

**Diamond, Rebecca.** 2016. "The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980–2000." *American Economic Review*, 106(3): 479–524.

**Diamond, Rebecca, and Cecile Gaubert.** 2022. "Spatial Sorting and Inequality." *Annual Review of Economics*, 14(1): 795–819.

**Dupuy, Arnaud, and Alfred Galichon.** 2014. "Personality Traits and the Marriage Market." *Journal of Political Economy*, 122(6): 1271–1319.

**Dupuy, Arnaud, Alfred Galichon, Sonia Jaffe, and Scott Duke Kominers.** 2020. "Taxation in Matching Markets." *International Economic Review*, 61(4): 1591–1634.

**Fernández, Raquel, Nezih Guner, and John Knowles.** 2005. "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality." *Quarterly Journal of Economics*, 120(1): 273–344.

**Galichon, Alfred, and Bernard Salanié.** 2022. "Cupid's Invisible Hand: Social Surplus and Identification in Matching Models." *Review of Economic Studies*, 89(5): 2600–2629.

**Gayle, George-Levi, and Andrew Shephard.** 2019. "Optimal Taxation, Marriage, Home Production, and Family Labor Supply." *Econometrica*, 87(1): 291–326.

**Golosov, Mikhail, and Ilia Krasikov.** 2025. "The Optimal Taxation of Couples." *Quarterly Journal of Economics*, 140(3): 2163–2211.

**Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante.** 2017. "Optimal Tax Progressivity: An Analytical Framework." *Quarterly Journal of Economics*, 132(4): 1693–1754.

**Hendren, Nathaniel.** 2020. "Measuring Economic Efficiency Using Inverse-Optimum Weights." *Journal of Public Economics*, 187, p. 104198.

**Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler.** 2022. "Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market." *American Economic Review*, 112(1): 169–212.

**Lockwood, Benjamin B., and Matthew C. Weinzierl.** 2016. "Positive and Normative Judgments Implicit in U.S. Tax Policy, and the Costs of Unequal Growth and Recessions." *Journal of Monetary Economics*, 77 30–47.

**McFadden, Daniel.** 1974. "Conditional Logit Analysis of Qualitative Choice Behavior." In *Frontiers in Econometrics*. ed. by Paul Zarembka, New York: Academic Press, 105–142.

**McFadden, Daniel.** 1978. "Modelling the Choice of Residential Location." In *Spatial Interaction Theory and Planning Models*. eds. by Anders Karlqvist, Lars Lundqvist, Folke Snickars, and Jörgen W. Weibull, Amsterdam: North-Holland, 75–96.

**McFadden, Daniel, and Kenneth Train.** 2000. "Mixed MNL Models for Discrete Response." *Journal of Applied Econometrics*, 15(5): 447–470.

**Nutz, Marcel.** 2022. "Introduction to Entropic Optimal Transport." Lecture notes, Columbia University, Version of December 5, 2022.

**Reed, William J., and Murray Jorgensen.** 2004. "The Double Pareto-Lognormal Distribution—A New Parametric Model for Size Distributions." *Communications in Statistics—Theory and Methods*, 33(8): 1733–1753.

**Resnick, Sidney I., and Rishin Roy.** 1991. "Random USC Functions, Max-Stable Processes and Continuous Choice." *Annals of Applied Probability*, 1(2): 267–292.

**Rudin, Walter.** 1991. *Functional Analysis*. New York: McGraw-Hill, , 2nd edition.

**Sachs, Dominik, Aleh Tsyvinski, and Nicolas Werquin.** 2020. "Nonlinear Tax Incidence and Optimal Taxation in General Equilibrium." *Econometrica*, 88(2): 469–493.

**Saez, Emmanuel.** 2001. "Using Elasticities to Derive Optimal Income Tax Rates." *Review of Economic Studies*, 68(1): 205–229.

**Scheuer, Florian, and Iván Werning.** 2017. "The Taxation of Superstars." *Quarterly Journal of Economics*, 132(1): 211–270.

**Shapley, Lloyd S., and Martin Shubik.** 1971. "The Assignment Game I: The Core." *International Journal of Game Theory*, 1(1): 111–130.

**Werning, Iván.** 2007. "Pareto Efficient Income Taxation." MIT Working Paper.

**Zeidler, Eberhard.** 1986. *Nonlinear Functional Analysis and its Applications I: Fixed-Point Theorems.* New York: Springer.

# Appendix

## 9 On preference shocks, expected utility, and matching

In this section, we relate the law of non-pecuniary shocks $\Psi_w$ to the expected utility $U_w$, state the regularity conditions used in our analysis, derive matching elasticities for specifications commonly used in applied work, and show that the matching equilibrium is unique.

### 9.1 Finite types

It is instructive to first examine the case of finite productivity types, i.e., $w \in W = \{w_1, ..., w_n\} \subset \mathbb{R}_{++}$. Recall the definition of expected utility in (2):

$$U_w(z) := \mathbb{E}_w \left[ \max_{w'} z(w') + \beta^{-1} \epsilon(w') \right],$$

where $z \in \mathbb{R}^n$ is a vector of pecuniary payoffs and $\epsilon \in \mathbb{R}^n$ is a random vector of preference shocks distributed according to $\Psi_w$ for a person with productivity $w$.

**Lemma 5.** *Suppose that, for each $w$, $\mathbb{E}_w|\epsilon(w')| < \infty$ for all $w'$. Suppose also that $\Psi_w$ admits a continuous density $\psi_w$ that is strictly positive on $\mathbb{R}^n$ and satisfies $\psi_w(\epsilon) \leq C_w(1 + \|\epsilon\|)^{-n}$ for some constant $C_w$. Then, for each $z$, there exists a (unique) pair $q_{w,z} \in \mathbb{R}^n_{++}$ and $\gamma_{w,z} \in \mathbb{R}^{n(n-1)}_{++}$ such that $q_{w,z}(w')\gamma_{w,z}(\tilde{w}'|w') = q_{w,z}(\tilde{w}')\gamma_{w,z}(w'|\tilde{w}')$ for all $\tilde{w}' \neq w'$, and*

$$U_w(z + dz) = U_w(z) + \sum_{w'} \left( 1 + \frac{\sum_{\tilde{w}' \neq w'} \gamma_{w,z}(\tilde{w}'|w')(dz(w') - dz(\tilde{w}'))}{2} \right) q_{w,z}(w') dz(w') + \mathrm{o}\left( \|dz\|^2_\infty \right) \tag{53}$$

*as $\|dz\|_\infty \to 0$. Furthermore, $(q_{w,z}, \gamma_{w,z})$ depend continuously on $z$.*

To simplify notation, we omit $z$ when referring to $q_w$ and $\gamma_w$ if no confusion can arise. In view of (53), $U_w$ is twice continuously differentiable in $z$, where $q_w$ and $\gamma_w$ capture its gradient and cross semi-elasticities, that is

$$q_w(w') = \frac{\partial U_w(z)}{\partial z(w')}, \quad \gamma_w(\tilde{w}'|w') = -\frac{\partial \ln q_w(w')}{\partial z(\tilde{w}')} \quad \forall \tilde{w}' \neq w'.$$

The symmetry condition on the product of $q_w$ and $\gamma_w$ is due to the fact that cross partials of expected utility with respect to $z$ agree. Both $q_w$ and $\gamma_w$ are strictly positive, vary continuously with $z$, and satisfy

$$\sum_{w'} q_w(w') = 1, \quad \sum_{\tilde{w}' \neq w'} \gamma_w(\tilde{w}'|w') = \frac{\partial \ln q_w(w')}{\partial z(w')} =: \bar{\gamma}_w(w'),$$

where $\bar{\gamma}_w(w')$ stands for the own semi-elasticity. The idea of the proof is standard and can be found for example in Galichon and Salanié (2022). Below, we provide a self-contained argument for completeness.

*Proof.* Since the shocks have finite means, $U_w$ is finite. Since they have a density, the maximizer in the definition of $U_w$ is almost surely unique. Hence, the maximand is almost surely differentiable in $z$, with gradient equal to the indicator of the maximizer, due to the envelope theorem. Then, the dominated convergence theorem implies that $U_w$ is differentiable with $\frac{\partial}{\partial z(w')}U_w(z)$ equal to the probability $q_w(w')$ that $w'$ is chosen, given by

$$q_w(w') = \Pr_w\left(z(w') + \beta^{-1}\epsilon(w') > z(\hat{w}') + \beta^{-1}\epsilon(\hat{w}') \;\; \forall \hat{w}' \neq w'\right). \tag{54}$$

Since the density of shocks is continuous and strictly positive on $\mathbb{R}^n$, the choice probability vector $q_w$ is strictly positive and depends continuously on $z$. To see that it is continuously differentiable, rewrite (54) more explicitly using the cumulative distribution function of the shocks $\Psi_w$ as

$$q_w(w') = \int \frac{\partial}{\partial \epsilon(w')} \Psi_w\left(1s + \beta(1z(w') - z)\right) ds,$$

where 1 is the $n$-dimensional vector of ones. By the bound on $\psi_w$ and the dominated convergence theorem, we can differentiate under the integral sign, and the resulting derivatives are continuous. Then, by the chain rule,

$$\gamma_w(\tilde{w}'|w') = \beta \frac{\int \frac{\partial^2}{\partial \epsilon(w')\partial \epsilon(\tilde{w}')} \Psi_w\left(1s + \beta(1z(w') - z)\right) ds}{q_w(w')} > 0 \;\; \forall \tilde{w}' \neq w',$$

and $\sum_{\tilde{w}' \neq w'} \gamma_w(\tilde{w}'|w') = \bar{\gamma}_w(w')$ follows from the fact that the choice probability vector is invariant to adding a constant to $z$. □

Lemma 5 shows how the law of shocks $\Psi_w$ defines the expected utility $U_w$. Remarkably, the lemma can be reversed. Specifically, assuming that $U_w$ admits a representation in (53), we can use (54) to reconstruct the density $\psi_w$ up to a normalization of one coordinate of the preference shock vector, i.e., we can identify the joint density of $(\epsilon_2 - \epsilon_1, ..., \epsilon_n - \epsilon_1)$, since the choice probabilities depend only on those pairwise differences.

Furthermore, under our regularity conditions listed in Lemma 5, we always have exactly one matching equilibrium.

**Lemma 6.** *The matching equilibrium exists and is unique.*

*Proof.* We characterize equilibria as the critical points of a strictly convex function, following Galichon and Salanié (2022). Consider the problem

$$\min_u \sum_w U_w(u_w) g(w)$$

subject to Condition (C), i.e., $u_w(w') + u_{w'}(w) = v(w, w')$. Since $\partial_{w'} U_w(u_w) = q_w(w')$, the first-order conditions for this convex problem are exactly Condition (B), i.e., $q_w(w')g(w) = q_{w'}(w)g(w')$. Since

each type can choose their own $w$ in the definition of $U_w$, we have

$$U_w(u_w) \geq \frac{1}{2}v(w,w) + \beta^{-1}\mathbb{E}_w\epsilon(w).$$

At the same time,

$$U_w(u_w) \geq \max_{w'} u_w(w') + \min_{w'} \beta^{-1}\mathbb{E}_w\epsilon(w').$$

Thus, it is without loss of generality to restrict attention to a compact set of $u$'s in the minimization problem, and existence of the matching equilibrium follows from the fact that the minimum is attained. To see that there exists exactly one equilibrium, note that, by Lemma 5, $U_w$ is strictly convex on the feasible set of $u$ satisfying $u_w(w) = \frac{1}{2}v(w,w)$. □

## 9.2 Continuous types

We now turn our attention to the model of continuous productivity types, $w \in W = [\underline{w}, \overline{w}] \subset \mathbb{R}_{++}$, that is the main focus of our paper. The arguments below apply to any compact $W \subset \mathbb{R}^k$, as in Section 7.1. We define expected utility $U_w(z)$ using the same "$\mathbb{E}_w \max[\cdot]$" formula, where $w \in W$ and pecuniary payoffs are captured by a continuous function $z : W \to \mathbb{R}$. As is standard, we equip spaces of continuous functions with the sup norm. In this formulation, preference shocks now constitute a so-called random field over $W$ (e.g., see Resnick and Roy 1991), and under appropriate regularity conditions on their law, one can exhibit the analogue of the lemma in the previous section. Since our main object of interest is expected utility itself, we take it as a primitive and state regularity conditions directly in terms of this functional. We refer the reader to Resnick and Roy (1991); Dagsvik (1994) for a general treatment of continuous choice, and to Resnick and Roy (1991); Dagsvik (1994) for extensive discussions of max-stable processes that generalize discrete multinomial logit models. For completeness, we sketch this generalization below.

**Assumption I.** *For each $(w,z)$, there exists a (unique) pair of continuous functions $q_{w,z} : W \to \mathbb{R}_{++}$ and $\gamma_{w,z} : W^2 \to \mathbb{R}_{++}$ so that $q_{w,z}(w')\gamma_{w,z}(\tilde{w}'|w') = q_{w,z}(\tilde{w}')\gamma_{w,z}(w'|\tilde{w}')$, and*

$$U_w(z+dz) = U_w(z) + \int \left(1 + \frac{\int \gamma_{w,z}(\tilde{w}'|w')(dz(w') - dz(\tilde{w}'))d\tilde{w}'}{2}\right) q_{w,z}(w')dz(w')dw' + \mathrm{o}\left(\|dz\|_\infty^2\right) \tag{55}$$

*as $\|dz\|_\infty \to 0$. Furthermore, $(q_{w,z}, \gamma_{w,z})$ depend continuously on $(w,z)$.*

As before, we omit $z$ when referring to $q$ and $\gamma$ if no confusion can arise. Condition (55) is a direct generalization of the representation in (53) to continuous types with the added continuity with respect to $w$, which is vacuously satisfied in finite-type settings. Under this condition, the expected utility is twice continuously differentiable in $z$, and $q$, $\gamma$ again capture its first and second derivatives. Specifically, $q$ is a strictly positive continuous density of choices and $\gamma$ corresponds to its cross semi-elasticities.

Given Assumption I made above, the matching equilibrium is unique, and the argument for uniqueness is identical to the one used in Lemma 6. Existence, however, requires additional conditions on the law of shocks, because even a bounded minimizing sequence need not have a convergent subsequence in the sup norm. Existence can be established for logit shocks; see Nutz (2022). For other laws of shocks with continuous types, we assume that the matching equilibrium exists.

We now provide several examples of models that fit this framework and whose discrete-type versions are used extensively in applied work.

**Example: logit** The continuous choice logit model was formalized in Dagsvik (1994) and then used to study matching in Dupuy and Galichon (2014). In this model, preference shocks take the form of a certain Poisson point process that gives

$$U_w(z) = \beta^{-1} \ln \left( \int e^{\beta z(w')} g(w') dw' \right).$$

It is easy to see that Assumption I holds with

$$q_w(w') = \frac{e^{\beta z(w')} g(w')}{\int e^{\beta z(\tilde{w}')} g(\tilde{w}') d\tilde{w}'}, \;\; \gamma_w(\tilde{w}'|w') = \beta q_w(\tilde{w}'),$$

and $\bar{\gamma}_w(w') = \beta$.

**Example: mixed logit** The logit model imposes rigid substitution patterns, i.e., $\gamma_w/q_w$ and $\bar{\gamma}_w$ are constant and equal to the scale parameter. One approach proposed in the literature to relax this is to add a layer of mixing (McFadden and Train 2000). Specifically, let $\theta$ be a (continuous) mixing parameter that is distributed with density $\mu$, and allow the type density $g_\theta$ to be a function of it so that expected utility takes the following form:

$$U_w(z) = \beta^{-1} \int \ln \left( \int e^{\beta z(w')} g_\theta(w') dw' \right) \mu(\theta) d\theta.$$

Assuming that all primitives are continuous and that $g_\theta$ is strictly positive, Assumption I holds with

$$q_w(w') = \int q_{w,\theta}(w') \mu(\theta) d\theta, \;\; \gamma_w(\tilde{w}'|w') = \beta \int \frac{q_{w,\theta}(w') q_{w,\theta}(\tilde{w}')}{q_w(w')} \mu(\theta) d\theta,$$

and $\bar{\gamma}_w(w') = \beta$, where $q_{w,\theta}(w') = \frac{e^{\beta z(w')} g_\theta(w')}{\int e^{\beta z(\tilde{w}')} g_\theta(\tilde{w}') d\tilde{w}'}$. This model allows for a bit more flexible cross substitution patterns while requiring the own semi-elasticity to be a constant.

**Example: FC-MNL** To gain even more flexibility than mixed logit, the so-called (mixed) nested logit is often employed. Alternatively, Davis and Schiraldi (2014) suggested the flexible coefficient multinomial logit (FC-MNL) model in which every pair of productivities is assigned a separate nest. Davis and Schiraldi (2014) show that FC-MNL is flexible enough to match any pattern of own- and

cross-price elasticities in discrete-choice settings. In our continuous choice setting, the FC-MNL model corresponds to the following form of expected utility:

$$U_w(z) = \beta^{-1} \ln \left( \int a(w', \tilde{w}') \left( \frac{e^{\frac{\beta}{\sigma} z(w')} + e^{\frac{\beta}{\sigma} z(\tilde{w}')}}{2} \right)^{\sigma} g(w') g(\tilde{w}') d(w', \tilde{w}') + \int b(w') e^{\beta z(w')} g(w') dw' \right),$$

where $\sigma \in (0, 1]$ controls the scale of within-pair substitution intensity, and $a, b \geq 0$ are continuous functions and $a$ is symmetric. Then, we have

$$\begin{aligned} q_w(w') &= \int \pi_w(\tilde{w}'|w') p(w', \tilde{w}') d\tilde{w}' + \bar{\pi}_w(w'), \\ \gamma_w(\tilde{w}'|w') &= \beta \frac{1-\sigma}{\sigma q_w(w')} \pi_w(\tilde{w}'|w') p(w', \tilde{w}') p(\tilde{w}', w') + \beta q_w(\tilde{w}'), \end{aligned}$$

where we used $p(w', \tilde{w}') = \frac{e^{\frac{\beta}{\sigma} z(w')}}{e^{\frac{\beta}{\sigma} z(w')} + e^{\frac{\beta}{\sigma} z(\tilde{w}')}}$, and

$$\pi_w(\tilde{w}'|w') = \frac{2a(w', \tilde{w}') \left( \frac{e^{\frac{\beta}{\sigma} z(w')} + e^{\frac{\beta}{\sigma} z(\tilde{w}')}}{2} \right)^{\sigma} g(w') g(\tilde{w}')}{e^{\beta U_w(z)}}, \quad \bar{\pi}_w(w') = \frac{b(w') e^{\beta z(w')} g(w')}{e^{\beta U_w(z)}}.$$

With $\sigma = 1$, FC-MNL reduces to the logit model.

### 9.3 Max-stable shocks and the multivariate extreme value (MEV) model

The logit and FC-MNL examples belong to the MEV model, which is extensively used in applied work. To see the connection, consider first the finite-type setting with $W = \{w_1, ..., w_n\}$. Recall (e.g., McFadden 1978 and Dagsvik 1994) that a distribution of shocks is said to be max-stable if its law admits the following representation:

$$\ln \Psi_w(\epsilon) = -L_w \left( e^{-\epsilon(w_1)}, ..., e^{-\epsilon(w_n)} \right).$$

This is precisely McFadden's MEV class, where $L_w \geq 0$ is homogeneous of degree one, unbounded in each coordinate, and satisfies the alternating sign condition $(-1)^{k+1} \partial^k L \geq 0$, where $\partial^k L$ is a distinct $k^{\text{th}}$ cross partial of $L$. The family of max-stable shocks (and thus the MEV model) is associated with the expected utility functional

$$U_w(z) = \beta^{-1} \ln L_w \left( e^{\beta z} \right),$$

up to the scaled Euler constant. Inspection of this representation reveals that finite-type versions of the logit and FC-MNL models satisfy this representation for certain $L_w$, e.g., $L_w(x) = \sum_{w'} x(w') g(w')$ for the logit; see also McFadden (1978) for other examples. The extension to continuous choice retains the same representation, with the only difference that $L_w$ is now a functional on positive continuous functions.

# 10 Proofs for Section 4

This section of the appendix collects the proofs of formal claims in Section 4 and related derivations. We maintain the conditions on expected utility stated in Sections 4 and 9, i.e., Assumption I, and make three additional sets of assumptions needed for our perturbational approach.

**Assumption II.**

- *The function $\mathcal{N}$ is twice continuously differentiable, strictly concave, symmetric, and $\mathcal{N}(l,l')/(l+l') \to -\infty$ as $l+l' \to \infty$.*
- *The tax $\mathcal{T}$ is twice continuously differentiable and symmetric, and its marginal tax rates are bounded below and strictly less than one.*
- *The tax function $\mathcal{T}$ is such that, for each $(w,w')$, problem (5) has a unique interior maximizer, and the Hessian of the objective with respect to $(l,l')$ is negative definite there.*

These conditions ensure that the optimal labor supplies are unique and vary smoothly when taxes are perturbed in any sufficiently smooth direction. Specifically, we consider tax reforms $\dot{\mathcal{T}}$ that are twice continuously differentiable, symmetric functions with bounded derivatives. Our conditions on $\mathcal{N}$ hold in the specifications of Sections 3 and 6, where $\mathcal{N}$ is quadratic or given by (38).

## 10.1 Linear operators

In our perturbational approach, we use various continuous linear operators mapping the space of continuous functions on $W^2$ to itself, where $W = [\underline{w}, \overline{w}] \subset \mathbb{R}_{++}$. We first collect some basic facts about such an operator $\mathbf{L}$ and then establish auxiliary results that are used in the proofs. By the Riesz representation theorem, each value $(\mathbf{L}\dot{\varphi})(w,w')$ can be represented as an integral of the continuous function $\dot{\varphi}$ against a finite signed measure on $W^2$. These measures may contain point masses represented by the Dirac delta function. The operator norm of $\mathbf{L}$ is

$$\|\mathbf{L}\| = \sup_{\dot{\varphi}:\|\dot{\varphi}\|_\infty=1} \|\mathbf{L}\dot{\varphi}\|_\infty,$$

and its spectral radius is given by $\lim_{k\to\infty} \|\mathbf{L}^k\|^{1/k}$, where $\mathbf{L}^k$ is the $k$-th application of $\mathbf{L}$. It is well known that, if this radius is strictly less than one, then $\mathbf{I}-\mathbf{L}$ is invertible, with

$$(\mathbf{I}-\mathbf{L})^{-1} = \sum_{k=0}^{\infty} \mathbf{L}^k,$$

where the series converges in operator norm and $\mathbf{L}^0 = \mathbf{I}$. These facts can be found, for example, in Rudin (1991).

Now, for a given tax function $\mathcal{T}$ and the associated matching equilibrium $(u, q)$, we consider $\mathbf{L} = \mathbf{A} - \mathbf{B}$, where the operators $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A}_{w,w'}(\tilde{w}, \tilde{w}') = \frac{\gamma_w(\tilde{w}'|w')}{\bar{\gamma}_w(w') + \bar{\gamma}_w^{\mathfrak{s}}(w')}\delta_{\tilde{w}=w}, \quad \mathbf{B}_{w,w'}(\tilde{w}, \tilde{w}') = \frac{\gamma_{w'}(\tilde{w}'|w)}{\bar{\gamma}_w(w') + \bar{\gamma}_w^{\mathfrak{s}}(w')}\delta_{\tilde{w}=w'}. \tag{56}$$

Since the matching elasticities used in their construction are continuous and strictly positive, $\mathbf{L}$ is an operator of the form described above. The following lemma establishes that its spectral radius is strictly less than one—the fact needed for Lemma 1—and that successive iterations shrink geometrically.

**Lemma 7.** *Let $\mathbf{L} = \mathbf{A} - \mathbf{B}$, and set $\gamma_* := \min_{(w,w',\tilde{w}')} \gamma_w(\tilde{w}'|w')$, $\bar{\gamma}^* := \max_{(w,w')} \bar{\gamma}_w(w')$. Then, the spectral radius of $\mathbf{L}$ is at most*

$$\rho = \sqrt{1 - \frac{1}{2}\left((\overline{w} - \underline{w})\gamma_*/\bar{\gamma}^*\right)^2} < 1,$$

*and $\|\mathbf{L}^{k+2}\| \leq \rho^2\|\mathbf{L}^k\|$ for every $k$.*

*Proof.* Both $\mathbf{A}$ and $\mathbf{B}$ are positive, and their combined weights sum to one, i.e.,

$$\int (\mathbf{A}_{w,w'}(\tilde{w}, \tilde{w}') + \mathbf{B}_{w,w'}(\tilde{w}, \tilde{w}'))d(\tilde{w}, \tilde{w}') = \int \frac{\gamma_w(\tilde{w}'|w') + \gamma_{w'}(\tilde{w}'|w)}{\bar{\gamma}_w(w') + \bar{\gamma}_w^{\mathfrak{s}}(w')}d\tilde{w}' = 1.$$

We can express the same statement in operator notation as $(\mathbf{A} + \mathbf{B})1 = 1$, where 1 stands for the constant function of $(w, w')$.

Take $\|\dot{\varphi}\|_\infty = 1$. Since the sup norm equals one, $1 \pm \dot{\varphi}$ is nonnegative, and we have

$$\begin{aligned} 1 - (\mathbf{L}\dot{\varphi}) &= \mathbf{A}(1 - \dot{\varphi}) + \mathbf{B}(1 + \dot{\varphi}) \\ &\geq \mathbf{B}(1 + \dot{\varphi}) \\ &\geq \frac{1}{2}(\gamma_*/\bar{\gamma}^*)\int (1 + \dot{\varphi}(w', \tilde{w}'))d\tilde{w}', \end{aligned}$$

where the first inequality is due to $\mathbf{A}(1 - \dot{\varphi}) \geq 0$, and the second is due to $1 + \dot{\varphi} \geq 0$ and the bounds on matching elasticities in the statement of the lemma. Similarly, we have

$$1 + \mathbf{L}\dot{\varphi} \geq \frac{1}{2}(\gamma_*/\bar{\gamma}^*)\int (1 - \dot{\varphi}(w', \tilde{w}'))d\tilde{w}'.$$

Now, using these two bounds once more, we can obtain

$$\begin{aligned} 1 - \mathbf{L}^2\dot{\varphi} &= \mathbf{A}(1 - \mathbf{L}\dot{\varphi}) + \mathbf{B}(1 + \mathbf{L}\dot{\varphi}) \\ &\geq \frac{1}{4}(\gamma_*/\bar{\gamma}^*)^2 \int (1 + \dot{\varphi}(\tilde{w}, \tilde{w}'))d(\tilde{w}, \tilde{w}') + \frac{1}{4}(\gamma_*/\bar{\gamma}^*)^2 \int (1 - \dot{\varphi}(\tilde{w}, \tilde{w}'))d(\tilde{w}, \tilde{w}') \\ &\geq \frac{1}{2}(\gamma_*/\bar{\gamma}^*)^2 \int d(\tilde{w}, \tilde{w}'), \end{aligned}$$

where the second line bounds each of the terms, $\mathbf{A}(1 - \mathbf{L}\dot{\varphi})$ and $\mathbf{B}(1 + \mathbf{L}\dot{\varphi})$, separately, and the third one uses the fact that $\dot{\varphi}$ cancels out. This shows that $\mathbf{L}^2\dot{\varphi} \leq \rho^2$ for any function $\dot{\varphi}$ with the unit norm, and applying the same argument to $-\dot{\varphi}$ gives the other bound, so that $\|\mathbf{L}^2\| \leq \rho^2$.

By continuity and compactness, $\gamma_* > 0$ and

$$\bar{\gamma}^* = \max_{(w,w')} \int \gamma_w(\tilde{w}'|w')d\tilde{w}' \geq \min_{(w,w',\tilde{w}')} \gamma_w(\tilde{w}'|w') \int d\tilde{w}' = (\overline{w} - \underline{w})\gamma_*,$$

which yields $\rho \in (0,1)$. Then, the spectral radius formula gives the first claim, and $\|\mathbf{L}^{k+2}\| \leq \|\mathbf{L}^2\|\|\mathbf{L}^k\|$ gives the second. □

## 10.2 Tax reforms in matching environments

We now consider perturbed taxes $\mathcal{T} + \varepsilon\dot{\mathcal{T}}$ and show that all equilibrium objects are well-defined for the tax reforms described above and vary smoothly for $\varepsilon$ near 0. We also prove Lemmas 1 and 3, which characterize the corresponding equilibrium responses. Following the convention introduced in the main text, we abbreviate all responses using dots, e.g., $\dot{v} := \frac{\partial}{\partial\varepsilon} v^\varepsilon\big|_{\varepsilon=0}$ stands for the response of the family surplus.

### 10.2.1 Behavioral effects and the proof of Lemma 3

We start with the perturbed problem (5), since its value, $v^\varepsilon$, is used as an ingredient in the perturbed matching equilibrium.

Our starting observation is that spousal labor supplies in that problem can be restricted to a compact set. Indeed, since marginal taxes are bounded below and the reform has bounded first derivatives, after-tax earnings grow at most linearly in total labor supply, uniformly over types and $\varepsilon$ sufficiently close to 0. To see this, let $\tau$ and $\dot{\tau}$ bound $\left|\frac{\partial\mathcal{T}}{\partial y}\right|$ and $\left|\frac{\partial\dot{\mathcal{T}}}{\partial y}\right|$, respectively. Then, for $|\varepsilon| \leq 1$, the objective in the perturbed problem (5) satisfies

$$wl + w'l' - \mathcal{T}(wl, w'l') - \varepsilon\dot{\mathcal{T}}(wl, w'l') + \mathcal{N}(l,l') \leq -\mathcal{T}(0,0) - \varepsilon\dot{\mathcal{T}}(0,0) + \overline{w}(1+\tau+\dot{\tau})(l+l') + \mathcal{N}(l,l').$$

By the growth condition in Assumption II, there exists a constant $C$, independent of types and $\varepsilon$, so that $l, l' = 0$ yields a strictly higher value of the objective than any other labor supplies with $l + l' \geq C$. Consequently, all maximizers exist and belong to the common compact set, $[0, C]^2$.

The first immediate implication is that the perturbed problem admits a solution, which is, in fact, unique for $|\varepsilon|$ small enough due to the uniqueness and the strict second-order condition at $\varepsilon = 0$ in Assumption II. Second, since the maximizer is unique, the envelope theorem applies, i.e., the family surplus is continuously differentiable in $\varepsilon$, and

$$\dot{v}(w,w') = -\dot{\mathcal{T}}(y_w(w'), y_w^{\mathfrak{s}}(w')).$$

Finally, the optimal earnings $y^\varepsilon$ are continuously differentiable in $\varepsilon$, with the response described in Lemma 3. To see this, first note that for all sufficiently small $|\varepsilon|$, due to Assumption II, the optimal

earnings in the perturbed problem are interior. Furthermore, the pair of optimal earnings $(y, y^{\mathfrak{s}}) = (y_w^{\varepsilon}(w'), y_w^{\varepsilon,\mathfrak{s}}(w'))$ satisfies the following first-order conditions:

$$1 - \frac{\partial}{\partial y}\mathcal{T}(y, y^{\mathfrak{s}}) - \varepsilon \frac{\partial}{\partial y}\dot{\mathcal{T}}(y, y^{\mathfrak{s}}) + \frac{1}{w}\frac{\partial}{\partial l}\mathcal{N}\left(\frac{y}{w}, \frac{y^{\mathfrak{s}}}{w'}\right) = 0, \tag{57}$$

$$1 - \frac{\partial}{\partial y}\mathcal{T}(y^{\mathfrak{s}}, y) - \varepsilon \frac{\partial}{\partial y}\dot{\mathcal{T}}(y^{\mathfrak{s}}, y) + \frac{1}{w'}\frac{\partial}{\partial l}\mathcal{N}\left(\frac{y^{\mathfrak{s}}}{w'}, \frac{y}{w}\right) = 0. \tag{58}$$

We can think of (57), (58) as a system of equations in $(y, y^{\mathfrak{s}})$ parameterized by $\varepsilon$. Due to Assumption II and twice continuous differentiability of $\dot{\mathcal{T}}$, the perturbed system is continuously differentiable in $(y, y^{\mathfrak{s}}, \varepsilon)$ in a neighborhood of the original optimal earnings and $\varepsilon = 0$.

Taking logs and totally differentiating these two conditions at $\varepsilon = 0$, we obtain

$$\begin{bmatrix} \left(\frac{y\frac{\partial^2\mathcal{T}}{\partial y^2}}{1-\frac{\partial\mathcal{T}}{\partial y}} - \frac{l\frac{\partial^2\mathcal{N}}{\partial l^2}}{-\frac{\partial\mathcal{N}}{\partial l}}\right) & \left(\frac{y^{\mathfrak{s}}\frac{\partial^2\mathcal{T}}{\partial y\partial y'}}{1-\frac{\partial\mathcal{T}}{\partial y}} - \frac{l^{\mathfrak{s}}\frac{\partial^2\mathcal{N}}{\partial l\partial l'}}{-\frac{\partial\mathcal{N}}{\partial l}}\right) \\ \left(\frac{y\frac{\partial^2\mathcal{T}^{\mathfrak{s}}}{\partial y\partial y'}}{1-\frac{\partial\mathcal{T}^{\mathfrak{s}}}{\partial y}} - \frac{l\frac{\partial^2\mathcal{N}^{\mathfrak{s}}}{\partial l\partial l'}}{-\frac{\partial\mathcal{N}^{\mathfrak{s}}}{\partial l}}\right) & \left(\frac{y^{\mathfrak{s}}\frac{\partial^2\mathcal{T}^{\mathfrak{s}}}{\partial y^2}}{1-\frac{\partial\mathcal{T}^{\mathfrak{s}}}{\partial y}} - \frac{l^{\mathfrak{s}}\frac{\partial^2\mathcal{N}^{\mathfrak{s}}}{\partial l^2}}{-\frac{\partial\mathcal{N}^{\mathfrak{s}}}{\partial l}}\right) \end{bmatrix} \begin{pmatrix} d\ln y \\ d\ln y^{\mathfrak{s}} \end{pmatrix} = \begin{pmatrix} -\frac{d\varepsilon}{1-\frac{\partial\mathcal{T}}{\partial y}}\frac{\partial\dot{\mathcal{T}}}{\partial y} \\ -\frac{d\varepsilon}{1-\frac{\partial\mathcal{T}^{\mathfrak{s}}}{\partial y}}\frac{\partial\dot{\mathcal{T}}^{\mathfrak{s}}}{\partial y} \end{pmatrix}, \tag{59}$$

where we suppress arguments and $(w, w')$ to simplify notation. By (57), (58) at $\varepsilon = 0$, $-\frac{\partial\mathcal{N}}{\partial l} = w\left(1 - \frac{\partial\mathcal{T}}{\partial y}\right)$. So the matrix on the left is minus the Hessian of the objective with respect to $(y, y^{\mathfrak{s}})$, with its rows divided by the retention rates and its columns multiplied by earnings. Since the second-order condition holds strictly at $\varepsilon = 0$, this Hessian is negative definite, so the matrix on the left has positive diagonal entries and a positive determinant. As a result, the implicit function theorem gives, for $\varepsilon$ near 0, exactly one solution $(y^{\varepsilon}, y^{\varepsilon,\mathfrak{s}})$ near the original optimal earnings, which is also continuously differentiable in $\varepsilon$. Furthermore, the response of the optimal earnings is given by (24), where

$$\begin{bmatrix} \xi & \zeta \\ \zeta^{\mathfrak{s}} & \xi^{\mathfrak{s}} \end{bmatrix}$$

is the inverse of the matrix on the left of (59). This shows the first part of Lemma 3.

To see the other parts, remark that

$$\xi = \frac{1}{\det}\left(\frac{y^{\mathfrak{s}}\frac{\partial^2\mathcal{T}^{\mathfrak{s}}}{\partial y^2}}{1-\frac{\partial\mathcal{T}^{\mathfrak{s}}}{\partial y}} - \frac{l^{\mathfrak{s}}\frac{\partial^2\mathcal{N}^{\mathfrak{s}}}{\partial l^2}}{-\frac{\partial\mathcal{N}^{\mathfrak{s}}}{\partial l}}\right), \quad \zeta = -\frac{1}{\det}\left(\frac{y^{\mathfrak{s}}\frac{\partial^2\mathcal{T}}{\partial y\partial y'}}{1-\frac{\partial\mathcal{T}}{\partial y}} - \frac{l^{\mathfrak{s}}\frac{\partial^2\mathcal{N}}{\partial l\partial l'}}{-\frac{\partial\mathcal{N}}{\partial l}}\right), \quad \zeta^{\mathfrak{s}} = -\frac{1}{\det}\left(\frac{y\frac{\partial^2\mathcal{T}^{\mathfrak{s}}}{\partial y\partial y'}}{1-\frac{\partial\mathcal{T}^{\mathfrak{s}}}{\partial y}} - \frac{l\frac{\partial^2\mathcal{N}^{\mathfrak{s}}}{\partial l\partial l'}}{-\frac{\partial\mathcal{N}^{\mathfrak{s}}}{\partial l}}\right),$$

where det is the determinant of that matrix. Then, the positivity of $\xi$ is due to the positive diagonal entries and $\det > 0$; the sign of $\zeta$ is that of the expression in Lemma 3 because $\det > 0$; and Slutsky symmetry follows from combining these expressions with (57), (58) evaluated at $\varepsilon = 0$ and using $\frac{\partial^2\mathcal{T}}{\partial y\partial y'} = \frac{\partial^2\mathcal{T}^{\mathfrak{s}}}{\partial y\partial y'}$, $\frac{\partial^2\mathcal{N}}{\partial l\partial l'} = \frac{\partial^2\mathcal{N}^{\mathfrak{s}}}{\partial l\partial l'}$.

### 10.2.2 Sorting and sharing effects and the proof of Lemma 1

We now turn to differentiability of the matching equilibrium. Conditions (A)–(C), with $v^{\varepsilon}$ replacing $v$, form a system of equations for the pair $(u, q)$ of continuous functions on $W^2$, which we equip with

the product sup norm. Due to Assumption I and the results above, this system is (Fréchet) continuously differentiable in $(u, q, \varepsilon)$ in a neighborhood of the unperturbed matching equilibrium and $\varepsilon = 0$. For $v^\varepsilon$, this uses that its derivative, $-\dot{\mathcal{T}}(y^\varepsilon_w(w'), y^{\varepsilon,\mathfrak{s}}_w(w'))$, is jointly continuous in $(w, w', \varepsilon)$. Totally differentiating (and taking logs in (A) and (B)), we obtain

$$\frac{dq_w(w')}{q_w(w')} = \int \gamma_w(\tilde{w}'|w')(du_w(w') - du_w(\tilde{w}'))d\tilde{w}', \quad \frac{dq_w(w')}{q_w(w')} = \frac{dq_{w'}(w)}{q_{w'}(w)}, \tag{60}$$

and $du_w(w') + du_{w'}(w) = \dot{v}(w, w')d\varepsilon$. Substituting $du_{w'}(w) = \dot{v}(w, w')d\varepsilon - du_w(w')$ into (60), we can express $(du, dq)$ using operator notation as in the main text:

$$(\mathbf{I} - \mathbf{A} + \mathbf{B})du = \mathbf{\Theta}\dot{v}d\varepsilon, \quad dq/q = \mathbf{C}du, \tag{61}$$

where $\mathbf{A}$ and $\mathbf{B}$ are defined in (56), and $\mathbf{C}$ and $\mathbf{\Theta}$ are given by

$$\begin{aligned}
\mathbf{C}_{w,w'}(\tilde{w}, \tilde{w}') &= \bar{\gamma}_w(w')\delta_{\tilde{w}=w}\delta_{\tilde{w}'=w'} - \gamma_w(\tilde{w}'|w')\delta_{\tilde{w}=w}, \\
\mathbf{\Theta}_{w,w'}(\tilde{w}, \tilde{w}') &= \frac{\bar{\gamma}^{\mathfrak{s}}_w(w')}{\bar{\gamma}_w(w') + \bar{\gamma}^{\mathfrak{s}}_w(w')}\delta_{\tilde{w}=w}\delta_{\tilde{w}'=w'}.
\end{aligned}$$

By Lemma 7, $\mathbf{I} - (\mathbf{A} - \mathbf{B})$ is invertible on the space of continuous functions on $W^2$, with a bounded inverse. The Banach-space implicit function theorem (e.g., Zeidler 1986) therefore gives that the perturbed matching equilibrium $(u^\varepsilon, q^\varepsilon)$ exists for each $\varepsilon$ sufficiently close to 0, and is (Fréchet) continuously differentiable in $\varepsilon$ in the product sup norm. Furthermore, by the uniqueness result in Section 9, it is the only perturbed matching equilibrium, and its derivative $(\dot{u}, \dot{q})$ solves the system above with $d\varepsilon = 1$.

Lemma 1 now follows from Lemma 7, $\dot{v} = -\dot{\mathcal{T}}$, and the fact that $\dot{f}/f = \dot{q}/q$ since $g$ does not change with the reform:

$$\dot{u} = (\mathbf{I} - \mathbf{A} + \mathbf{B})^{-1}\mathbf{\Theta}\dot{v} = -\mathbf{K}\dot{\mathcal{T}}, \quad \dot{f}/f = \dot{q}/q = \mathbf{C}\dot{u} = -\mathbf{M}\dot{\mathcal{T}}.$$

### 10.2.3 Proof of Lemma 2

Suppose first that $\dot{f} = 0$, which is equivalent to $\dot{q} = 0$ since $g$ does not change with the reform. For each $w$, take a maximizer $w'$ of $\dot{u}_w$, so that

$$0 = \int \gamma_w(\tilde{w}'|w') \left(\dot{u}_w(w') - \dot{u}_w(\tilde{w}')\right) d\tilde{w}',$$

due to (60). Since $\gamma_w > 0$ and, at a maximizer, every difference $\dot{u}_w(w') - \dot{u}_w(\tilde{w}')$ is nonnegative, the only way this equation can hold is that $\dot{u}_w(\tilde{w}') = \dot{u}_w(w')$ for all $\tilde{w}' \neq w'$ (by continuity of $\dot{u}_w$ with continuous types). That is, the change in pecuniary utility is independent of the partner's productivity. The envelope theorem and feasibility now imply

$$-\dot{\mathcal{T}}(w, w') = \dot{v}(w, w') = \dot{u}_w(w) + \dot{u}_{w'}(w'),$$

which makes the reform additively separable, with $\dot{\tau}(w) = -\dot{u}_w(w)$.

Conversely, suppose that $\dot{\mathcal{T}}(w,w') = \dot{\tau}(w)+\dot{\tau}(w')$. The response $\dot{u}_w(w') = -\dot{\tau}(w)$ satisfies feasibility and, since it does not vary across partners, leaves every choice probability unchanged. It therefore satisfies both equations in (60) with $\dot{q} = 0$. By the uniqueness established in the proof of Lemma 1, $(\dot{u}, \dot{q}) = (-\dot{\tau}, 0)$, and thus $\dot{f} = 0$.

For part (a), a uniform lump-sum tax or transfer $\dot{\mathcal{T}} = \pm 1$ is separable with $\dot{\tau} = \pm\frac{1}{2}$. Thus $\dot{f} = 0$ and $\dot{u} = \mp\frac{1}{2}$.

For part (b), absence of complementarities and symmetry give $\mathcal{N}(l,l') = -N(l) - N(l')$ for some function $N$. With individual-based taxes, the maximization problem (5) separates into the two spouses' individual problems, so earnings depend only on own type. The statutory tax change is therefore additively separable.

## 10.3 Individual- and family-based taxation of income

Below, we prove the results that appear in Section 5. We focus only on family-based taxation since the arguments for individual-based taxation are identical, except for Proposition 5, which we prove at the end. Specifically, the proofs of Propositions 3 and 4 directly follow those of Propositions 1 and 2, respectively.

### 10.3.1 Welfare-maximizing family-based taxes and the proof of Proposition 1

First of all, remark that since family earnings $Y$ are continuously differentiable in types with a nonvanishing gradient (differentiating (57), (58) evaluated at $\varepsilon = 0$ with respect to $(w,w')$ shows that the Jacobian of $(y, y^{\mathbf{s}})$ has a positive determinant), and $f$, $\xi$, and $\zeta$ are continuous, the density of family earnings $h^{\mathrm{F}}$ and the elasticity of family earnings $\bar{\xi}^{\mathrm{F}}$ are (right-)continuous.[26] Let $\left[\underline{Y}, \overline{Y}\right]$ be the range of family earnings. Writing a reform as $\dot{T}^{\mathrm{F}}(Y) = \dot{T}^{\mathrm{F}}(\underline{Y}) + \int_{\underline{Y}}^{\overline{Y}} \frac{\partial}{\partial Y}\dot{T}^{\mathrm{F}}(t)1_{Y>t}dt$ and substituting into (28) and (29), we obtain

$$\dot{W} + \dot{R} = \int_{\underline{Y}}^{\overline{Y}} \left(\frac{1}{2}\Delta^{\mathrm{F}}(t) - (1-H^{\mathrm{F}}(t))\mathbb{E}[\mathbf{K}^*\alpha|Y>t]\right)\frac{\partial}{\partial Y}\dot{T}^{\mathrm{F}}(t)dt, \tag{62}$$

where the terms proportional to $\dot{T}^{\mathrm{F}}(\underline{Y})$ cancel because

$$\int \mathbf{K}^*\alpha dF = \int \alpha\cdot\mathbf{K}1dF = \frac{1}{2}, \quad \int \mathbf{M}^*T^{\mathrm{F}}dF = \int T^{\mathrm{F}}\cdot\mathbf{M}1dF = 0$$

by Lemma 2. Therefore, since the term in the round brackets in (62) is (right-)continuous, (31) holds if and only if (32) holds at every $t$, due to the fundamental lemma of the calculus of variations. Hence, substituting (30) into (32) and dividing by $t\bar{\xi}^{\mathrm{F}}(t)h^{\mathrm{F}}(t)$, which is positive at interior $t$, gives (33), and the proposition follows.

[26] In principle, even under our assumptions, the density of earnings $h^{\mathrm{F}}$ can have jumps, yet it always admits a right-continuous representative.

### 10.3.2 Efficient family-based taxes and the proof of Proposition 2

For a probability measure $\alpha$ on $W^2$, we read the left-hand side of (32) as $\int \mathbf{K}1_{Y>t}\,d\alpha$. This equals $(1-H^{\mathrm{F}}(t))\mathbb{E}[\mathbf{K}^*\alpha|Y>t]$ when $\alpha$ has a density with respect to $F$. Here, $\mathbf{K}$ acts on bounded measurable functions, such as $1_{Y>t}$, through the same series (20), which converges by the argument of Lemma 7, and Fubini's theorem lets it pass under the integral over $t$. Repeating the argument in the proof of Proposition 1 with $\int \dot{u}\,d\alpha$ in place of $\dot{W}$ then gives, for every family-based reform,

$$\int \dot{u}d\alpha + \dot{R} = \int_{\underline{Y}}^{\overline{Y}} \left(\frac{1}{2}\Delta^{\mathrm{F}}(t) - \int \mathbf{K}1_{Y>t}d\alpha\right) \frac{\partial}{\partial Y}\dot{T}^{\mathrm{F}}(t)dt. \tag{63}$$

Let $T^{\mathrm{F}}$ be not efficient and let $\dot{T}^{\mathrm{F}}$ be a Pareto improvement, i.e., $\dot{R} \geq 0$ and $\dot{u} > 0$ in all matches. Towards a contradiction, suppose that there exist Pareto weights $\alpha$—a probability measure on $W^2$—satisfying the conditions of the Proposition. Then, by (63),

$$\int \dot{u}d\alpha + \dot{R} = 0.$$

However, $\int \dot{u}d\alpha > 0$ since $\dot{u} > 0$ and $\alpha$ is a probability measure, and $\dot{R} \geq 0$, a contradiction.

Conversely, let $T^{\mathrm{F}}$ be efficient. The pairs $(\dot{u}, \dot{R})$ generated by family-based reforms, i.e.,

$$\mathcal{S} = \left\{(\dot{u}, \dot{R})|\dot{T}^{\mathrm{F}} \text{ is twice continuously differentiable with bounded derivatives}\right\},$$

where $(\dot{u}, \dot{R})$ are given by (21) and (29), form a linear subspace of the space of continuous functions on $W^2$ times $\mathbb{R}$. Since taxes are efficient, the intersection of $\mathcal{S}$ and the set of pairs with $\dot{u} > 0$ and $\dot{R} > 0$ is empty. The first set is convex, and the second is convex and open since $W$ is compact. Therefore, by the Hahn–Banach separation theorem (e.g., Rudin 1991, Theorem 3.4) and the Riesz representation theorem, there exist a Borel measure $\alpha \geq 0$ on $W^2$ and a constant $c \geq 0$, not both zero, such that

$$\int \dot{u}d\alpha + c\dot{R} = 0$$

for every point of $\mathcal{S}$. Both are nonnegative because the separating functional is positive on that open set. By Lemma 2, the lump-sum reform $\dot{T}^{\mathrm{F}} = 1$ has $\dot{u} = -\frac{1}{2}$ and $\dot{R} = \frac{1}{2}$, so $c = \int d\alpha$, and hence $c > 0$, since $\alpha$ and $c$ are not both zero.

Thus, dividing $\alpha$ by $c$, we obtain a probability measure on $W^2$, still denoted by $\alpha$, for which the left-hand side of (63) vanishes for every reform. The fundamental lemma of the calculus of variations then gives (32) almost everywhere. Since $t \mapsto \int \mathbf{K}1_{Y>t}\,d\alpha$ is right-continuous by dominated convergence, and so is $\Delta^{\mathrm{F}}$ by the right-continuity of $h^{\mathrm{F}}$ and $\bar{\xi}^{\mathrm{F}}$, (32) holds at every $t$. Substituting (30) as in the proof of Proposition 1 gives the equivalent form (33), with $(1 - H^{\mathrm{F}}(t))\mathbb{E}[\mathbf{K}^*\alpha|Y > t]$ read as above.

### 10.3.3 Proof of Proposition 5

Since there are no complementarities and taxes are individual-based, examination of the first-order conditions (57), (58) at $\varepsilon = 0$ and of the matrix in (59) reveals that the optimal earnings $y_w(w')$ and the own elasticity $\xi_w(w')$ are independent of the partner's productivity. As a result, every reform is additively separable. By Lemma 2, $\dot{f} = 0$ and $\dot{u} = -\dot{T}$, i.e.,

$$\mathbf{K}\dot{\mathcal{T}} = \dot{T}, \;\; \mathbf{M}\dot{\mathcal{T}} = 0,$$

so tax perturbations do not affect sorting. Thus, the sorting term in (25) equals zero, and, by (28), $\dot{W} = -\int \alpha \dot{T} dF$. Furthermore, the cross-elasticity $\zeta_w(w')$ is zero, so that the indirect behavioral effect in (25) is zero. Putting all pieces together,

$$\dot{W} + \dot{R} = \int (1-\alpha)\dot{T} dF - \int \frac{y\xi \frac{\partial T}{\partial y}}{1 - \frac{\partial T}{\partial y}} \frac{\partial \dot{T}}{\partial y} dF,$$

and using the fundamental lemma of the calculus of variations as in the proof of Proposition 1, we obtain the formula in the proposition, which is the optimal tax formula of Diamond (1998), as shown in Saez (2001).

# 11 Quantitative illustration

In this section, we detail our quantitative illustration in Section 6. We organize our discussion into two subsections: calibration and numerical implementation.

## 11.1 Calibration

Our quantitative model has three sets of parameters: those of the (dis)utility of labor supply, those of the distribution of productivity, and the parameter $\beta$, which captures the importance of non-pecuniary shocks, since these shocks are assumed to follow the logit model. We calibrate these parameters sequentially, in this order.

### 11.1.1 (Dis)utility of labor supply

We consider $\mathcal{N}$ of the form (38), parameterized by $\sigma$ and $\varrho$, which control the elasticity of labor supply and complementarities, respectively. It is strictly concave whenever $\chi := (1+\varrho)(1+1/\sigma) - 1 > 0$. Direct calculations show that for linear taxes with $\frac{\partial \mathcal{T}}{\partial y} = \tau$, the optimal earnings $y_w(w')$ in (5) are given by

$$y_w(w') = (1-\tau)^\sigma w^{1+1/\chi} \left( \frac{(w)^{1+1/\chi} + (w')^{1+1/\chi}}{1+1/\sigma} \right)^{\frac{\sigma\varrho}{1+\varrho}},$$

so that the individual's share of family earnings is $\mathrm{s} = w^{1+1/\chi}/(w^{1+1/\chi} + w'^{1+1/\chi})$, and family earnings are proportional to $(1-\tau)^\sigma$. Under linear taxes, the tax terms in the matrix on the left of (59) vanish, and the matrix becomes

$$\begin{bmatrix} \chi - \varrho(1+1/\sigma)\mathrm{s} & -\varrho(1+1/\sigma)(1-\mathrm{s}) \\ -\varrho(1+1/\sigma)\mathrm{s} & \chi - \varrho(1+1/\sigma)(1-\mathrm{s}) \end{bmatrix},$$

with determinant $\chi/\sigma$. Inverting it gives the own- and cross-elasticities in (39). Since $F$ is symmetric, the population average of s equals $\frac{1}{2}$, and averaging (39) gives

$$\mathbb{E}\xi + \mathbb{E}\zeta = \sigma, \quad \mathbb{E}\xi - \mathbb{E}\zeta = \frac{1}{\chi}, \tag{64}$$

where $\mathbb{E}\xi$ and $\mathbb{E}\zeta$ are the average values of behavioral elasticities. To obtain $\mathbb{E}\xi$ and $\mathbb{E}\zeta$, we rely on the empirical estimates of Calvo et al. (2024) that we detail below.

### 11.1.2 Calibration of $\mathbb{E}\xi$ and $\mathbb{E}\zeta$

Calvo et al. (2024) develop and estimate a bilateral matching model of the marriage market in which complementarities in the production of the public good determine sorting and sharing, using data from the German Socio-Economic Panel (GSOEP). Their model of households can be seen as a gender-specific instance of the setting in Example 1. As in Section 12.1.1, we use a prime "′" to denote a variable of the person from the second group—female for concreteness. Then, the model studied and estimated in Calvo et al. (2024) can be expressed as in (7) with $\kappa = 1$, i.e., the home good is fully public, and with the production technology

$$D(m, m') = 38.33 \times \left(0.18(m)^{-0.54} + 0.82(m')^{-0.54}\right)^{-1/0.54},$$

which is asymmetric, capturing the fact that women are relatively more productive at home. The cost function is simply $N(x) = 0$ if $x \leq 1$ and $N(x) = \infty$ if $x > 1$, meaning that each person is free to allocate one unit of time between labor and home at no cost. Finally, the function $\mathcal{T}$ is of the form

$$\mathcal{T}(y, y') = \left[(y) - 6.32 - 42\,(y)^{0.63}\right] + \left[(y') - 6.32 - 42\,(y')^{0.63}\right].$$

In Calvo et al. (2024), this function is interpreted as the gap between a worker's effective labor $y$ and their market earnings, which equal $6.32 + 42y^{0.63}$, rather than a tax function chosen by the government. From the households' point of view, whether $\mathcal{T}$ is part of the labor market technology or a tax is immaterial; thus the analysis of behavioral elasticities is identical to that in Section 4.2.

Calvo et al. (2024) report the equilibrium matching in the data conditional on spousal educational levels. Each person has one of three education levels—low, medium, high—and we discretize labor market productivity for each group into three values: $w \in W = \{w_l, w_m, w_h\}$ and $w' \in W' = \{w'_l, w'_m, w'_h\}$.

| | $w'_l$ | $w'_m$ | $w'_h$ |
|---|---|---|---|
| $w_l$ | 0.1306 | 0.0895 | 0.0116 |
| $w_m$ | 0.0667 | 0.1850 | 0.0328 |
| $w_h$ | 0.0188 | 0.0718 | 0.0846 |

Table 1: Matching frequencies by male productivity category (rows) and female productivity category (columns), taken from Table 3 of Calvo et al. (2024), excluding singles.

We infer these productivities from the regression coefficients reported in Table O.12 of their supplementary appendix as follows. We set $w_m$ and $w_h$ equal to the exponentiated coefficients on the dummies for medium and high education in their regression of log hourly wages (regression 3), normalize $w_l = 1$ since the constant of that regression is not reported, and multiply female productivities by their estimated gender productivity gap 0.85, which gives

$$w_l = 1,\ w_m \approx 1.03,\ w_h \approx 1.22,$$

and $w' = 0.85w$ for each education level. The equilibrium matching in the data, as documented by Calvo et al. (2024), is reported in Table 1, and we normalize these frequencies to sum to one to obtain $f(w, w')$.

To obtain behavioral elasticities, we consider two reforms, one for each gender. First, suppose that taxes become

$$\tilde{\mathcal{T}}(y, y') = \mathcal{T}(y, y') + \tau_m \left(6.32 + 42\,(y)^{0.63}\right),$$

where $\tau_m = 0.01$. We solve for the optimal spousal earnings $(\tilde{y}, \tilde{y}')$ under these perturbed taxes and approximate two behavioral elasticities from the log changes in earnings: the own male elasticity

$$\xi_w(w') \approx \frac{\ln \tilde{y} - \ln y}{\ln(1 - \tau_m)}$$

and the cross female elasticity

$$\zeta_{w'}(w) \approx \frac{\ln \tilde{y}' - \ln y'}{\ln(1 - \tau_m)}.$$

We then repeat the exercise for taxes on women and obtain two additional elasticities: the own female elasticity $\xi_{w'}(w)$ and the cross male elasticity $\zeta_w(w')$. Finally, using the matching, we compute the following two ratios of cross-to-own elasticities:

$$\frac{\sum_{w,w'} \zeta_w(w') f(w, w')}{\sum_{w,w'} \xi_w(w') f(w, w')} \approx \frac{0.138}{0.295} \approx 0.47,\ \ \frac{\sum_{w,w'} \zeta_{w'}(w) f(w, w')}{\sum_{w,w'} \xi_{w'}(w) f(w, w')} \approx \frac{0.286}{1.072} \approx 0.27.$$

These two numbers are the source of "between one quarter and one half" in Section 6.1 and of the target $\mathbb{E}\zeta = \frac{1}{4}\mathbb{E}\xi$.

### 11.1.3 Marginal distribution of productivity

To calibrate $G$, following Golosov and Krasikov (2025), we first recover each spouse's productivity from the observed earnings, measured in hundreds of thousands of dollars, using the inversion in Section 6. We then fit a Pareto-lognormal distribution (Reed and Jorgensen, 2004),

$$G(w) = \Phi\left(\frac{\ln w - \eta}{\nu}\right) - e^{a\eta + \frac{a^2\nu^2}{2}} w^{-a} \Phi\left(\frac{\ln w - \eta - a\nu^2}{\nu}\right),$$

whose upper tail is Pareto with index $a$, i.e., $1 - G(w) \sim \text{const} \times w^{-a}$ as $w \to \infty$. We estimate $a$ by regressing $\ln(1 - \hat{G}(w))$ on $\ln w$ between the 90th and 99th percentiles of the empirical distribution $\hat{G}$, which gives $a \approx 3.56$. We exclude the top percentile because of sampling noise. We then choose $\eta$ and $\nu$ to match the mean and the Gini coefficient, which have the following closed forms (see, e.g., Colombi, 1990):

$$\mathbb{E}w = \frac{a}{a-1} e^{\eta + \frac{\nu^2}{2}}, \quad \text{Gini} = 2\Phi\left(\frac{\nu}{\sqrt{2}}\right) - 1 + \frac{2e^{a(a-1)\nu^2}}{2a-1} \Phi\left(\frac{(1-2a)\nu}{\sqrt{2}}\right).$$

In our sample, these statistics are 0.90 and 0.30, which gives $\eta \approx -0.53$ and $\nu \approx 0.44$. Figure 4 compares the empirical and fitted distribution functions. For the benchmark without complementarities, the earnings share drops out of the inversion, and the same procedure gives $a \approx 3.39$, $\eta \approx -0.63$, and $\nu \approx 0.44$.

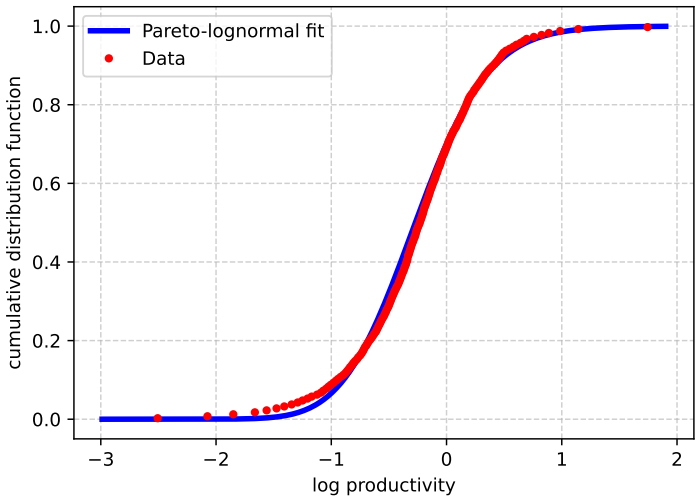



Figure 4: Empirical and fitted Pareto-lognormal distribution functions of log productivity.

### 11.1.4 Importance of non-pecuniary shocks

To calibrate $\beta$, we use Kendall's tau of spouses' productivities,

$$\Pr\left((w - \tilde{w})(w' - \tilde{w}') > 0\right) - \Pr\left((w - \tilde{w})(w' - \tilde{w}') < 0\right),$$

where $(w, w')$ and $(\tilde{w}, \tilde{w}')$ are independent draws from $F$. Since $F$ has no closed form, for each $\beta$ we compute the equilibrium under the linear tax on the grid described in Section 11.2, evaluate Kendall's tau of the resulting discrete distribution by counting concordant minus discordant pairs of grid points, and solve for $\beta$ with a root finder. In our sample, Kendall's tau of the recovered productivities equals 0.10, which yields $\beta \approx 33$. In the benchmark without complementarities, matching is random for every $\beta$, so this parameter plays no role.

## 11.2 Numerical implementation

In our numerical approach, we fix the earnings grid $\{y_i\}_{i=1}^{N_y}$ of 400 points, equally spaced in logs between 0.05 and 80, i.e., between \$5,000 and \$8 million. The algorithm proceeds in five consecutive steps, which are repeated, starting from $\tau_i = 0.3$, until the optimal tax function is found.

**Step 1: constructing the tax function** Given marginal rates $\{\tau_i\}_{i=1}^{N_y}$, we construct $T$ by the trapezoidal rule in logs,

$$T(y_i) = \sum_{j=2}^{i} \frac{\tau_j y_j + \tau_{j-1} y_{j-1}}{2} (\ln y_j - \ln y_{j-1}),$$

normalized by $T(y_1) = 0$, and interpolate it with a cubic spline. Separately, we interpolate tax rates $\frac{\partial T}{\partial y}$ using a cubic spline of $\{\ln(1-\tau_i)\}_{i=1}^{N_y}$ in $\{\ln y_i\}_{i=1}^{N_y}$. This approach keeps interpolated rates below one, and the derivative of the spline delivers $\frac{y \partial^2 T/\partial y^2}{1-\partial T/\partial y}$, which enters (59).

**Step 2: assembling the productivity grid** Given the tax function $T$, let

$$\mathrm{s}(y, y') := \frac{\left(1 - \frac{\partial}{\partial y} T(y)\right) y}{\left(1 - \frac{\partial}{\partial y} T(y)\right) y + \left(1 - \frac{\partial}{\partial y} T(y')\right) y'}$$

be the share of after-tax marginal earnings of a person earning $y$ married to a person earning $y'$, and let

$$\hat{w}(y, y') := \frac{y^{\frac{1}{1+\sigma}}}{\left(1 - \frac{\partial}{\partial y} T(y)\right)^{\frac{1}{1+1/\sigma}}} \left((1 + 1/\sigma)\mathrm{s}(y, y')\right)^{\frac{\varrho}{(1+\varrho)(1+1/\sigma)}}$$

be the productivity of this person, obtained by inverting the first-order conditions (57), (58) at $\varepsilon = 0$. This is the inversion of Section 6 with own and spouse marginal rates.

We define the productivity grid $W = \{w_i\}_{i=1}^{N_w}$ of 400 points, equally spaced in logs between the minimum and the maximum of $\{\hat{w}(y_i, y_j)\}_{i,j=1}^{N_y}$. Each grid point $w_i$ receives the mass $g(w_i) = G(w_i e^{\Delta/2}) - G(w_i e^{-\Delta/2})$, where $\Delta$ is the log spacing, and the masses of the two tails are added to the end points, $w_1$ and $w_{N_w}$. At this stage, we also renormalize the Pareto weights on $W$ so that $\sum_{i=1}^{N_w} \alpha(w_i) g(w_i) = 1$.

**Step 3: solving the couples' problem** Next, we solve for the optimal earnings $y_w(w')$ for each pair $w, w' \in W$. They are characterized by the condition

$$\hat{w}\big(y_w(w'), y_{w'}(w)\big) = w \;\; \forall w, w' \in W,$$

which we solve by damped fixed-point iterations in log earnings. Starting from $y_{w_i}^{(0)}(w_j) = y_i$, we iterate

$$\ln y_w^{(n+1)}(w') = \ln y_w^{(n)}(w') - \frac{1}{2}\Big(\ln \hat{w}\big(y_w^{(n)}(w'), y_{w'}^{(n)}(w)\big) - \ln w\Big)$$

until the largest change is below a tolerance threshold.

Having found the optimal earnings on $W^2$, we substitute them into the objective in (5) to obtain the family surplus on $W^2$, and into the left-hand side of (59) to find the behavioral elasticities on $W^2$. Specifically, for (38), we have

$$\begin{bmatrix} \xi & \zeta \\ \zeta^{\mathfrak{s}} & \xi^{\mathfrak{s}} \end{bmatrix} = \begin{bmatrix} \chi - \varrho(1+1/\sigma)\mathrm{s} + \frac{y\,\partial^2 T/\partial y^2}{1-\partial T/\partial y} & -\varrho(1+1/\sigma)(1-\mathrm{s}) \\ -\varrho(1+1/\sigma)\mathrm{s} & \chi - \varrho(1+1/\sigma)(1-\mathrm{s}) + \frac{y^{\mathfrak{s}}\,\partial^2 T^{\mathfrak{s}}/\partial y^2}{1-\partial T^{\mathfrak{s}}/\partial y} \end{bmatrix}^{-1},$$

where $\mathrm{s} = \mathrm{s}(y, y^{\mathfrak{s}})$, and the arguments and the dependence on $(w, w')$ are suppressed to simplify notation.

**Step 4: finding the matching equilibrium** To find the matching equilibrium, we follow the approach outlined in the main text. With $\delta_i := U_{w_i}(u_{w_i}) - \frac{1}{2}v(w_i, w_i)$, equation (41) becomes a fixed point in $\{\delta_i\}_{i=1}^{N_w}$ whose exponents stay bounded at large $w_i$. We solve it by damped fixed-point iterations starting from $\delta = 0$. We then recover $q$ from (40), normalize each row to sum to one, and set $f(w_i, w_j) = q_{w_i}(w_j)g(w_i)$.

**Step 5: computing the effects of taxation and updating taxes** With logit shocks, the adjoints in (36) have closed forms in terms of the matrix $\mathbf{P} = (\mathbf{I} + \mathbf{Q})^{-1}$ of Section 6. Using

$$p_{\tilde{w}}(w) + \sum_{\tilde{w}'} p_{\tilde{w}'}(w) q_{\tilde{w}}(\tilde{w}') = 1_{\tilde{w}=w},$$

which follows from $(\mathbf{I} + \mathbf{Q})\mathbf{P} = \mathbf{I}$, we obtain

$$\begin{aligned} (\hat{\mathbf{K}}^* \alpha)(w, w') &= \frac{1}{g(w)} \sum_{\tilde{w}} \alpha(\tilde{w}) g(\tilde{w}) p_{\tilde{w}}(w), \\ (\hat{\mathbf{M}}^* \mathcal{T})(w, w') &= \frac{\beta}{2}\mathcal{T}(w, w') - \frac{\beta}{g(w)} \sum_{\tilde{w}, \tilde{w}'} \mathcal{T}(\tilde{w}, \tilde{w}') q_{\tilde{w}}(\tilde{w}') g(\tilde{w}) p_{\tilde{w}}(w), \end{aligned}$$

which capture the sharing and sorting effects in decomposition (37).

Next, we translate all effects to the original earnings grid. For any function $X$ on $W^2$, $(1 - H(t))\mathbb{E}[X|y > t] = \sum_{w,w'} X(w, w') 1_{y_w(w') \geq t} f(w, w')$ is computed exactly. Conditional means at $y = t$, namely $\bar{\xi}(t)$ and the indirect behavioral term, are computed with a Nadaraya–Watson estimator with

a Gaussian kernel in log earnings. The hazard ratio $\frac{1-H(t)}{h(t)t}$ is estimated by a local linear regression of $\ln(1-H)$ on $\ln t$. We add a bit of smoothing to all variables to ensure that the discretization noise does not affect the resulting tax rates. Finally, we compute the right-hand side of (36), set $\bar{\tau}_i/(1-\bar{\tau}_i)$ equal to it to obtain the implied rates $\bar{\tau}_i$, update rates as a convex combination of $\tau_i$ and $\bar{\tau}_i$, and stop when the largest change is below a tolerance level.

# 12 Taxation in other matching environments

In this section, we explain how our analysis of the baseline economy extends to other settings with endogenous matching and provide auxiliary results mentioned in the main text. To streamline the exposition, we focus on the underlying economics, assuming that the setting is sufficiently regular, e.g., that the matching equilibrium is unique and differentiable, as in the baseline economy under Assumptions I and II.

## 12.1 Heterogeneity, asymmetries, and outside options

### 12.1.1 Bilateral matching

Suppose that individuals belong to two groups, each of mass $\frac{1}{2}$, and match only with the other group. We distinguish objects of the second group with a prime "′". Within each group, types $w$ and $w'$ have probability distributions $G$ and $G'$, with densities $g$ and $g'$. The joint distribution of matches $F$, defined below, therefore has mass one, while the mass of couples is $\frac{1}{2}$. The groups may have different laws of non-pecuniary shocks, and thus different expected utility functionals $U_w$ and $U'_{w'}$, defined as in (2), and neither $\mathcal{N}$ nor $\mathcal{T}$ need be symmetric. The family surplus $v$ is still given by (5), where $l$ and $l'$ are the labor supplies of the first- and second-group spouses.

**Definition.** An equilibrium in the marriage market given the family surplus $v$ consists of functions $(u, q)$ and $(u', q')$ such that

(A). $q_w = \partial U_w(u_w)$ and $q'_{w'} = \partial U'_{w'}(u'_{w'})$;

(B). $q_w(w')g(w) = q'_{w'}(w)g'(w')$;

(C). $u_w(w') + u'_{w'}(w) = v(w, w')$.

The joint density of matches is $f(w, w') := q_w(w')g(w) = q'_{w'}(w)g'(w')$. Adding the same constant to all first-group utilities and subtracting it from all second-group utilities leaves these conditions unchanged. We fix the constant by requiring $\int U_w(u_w)dG = \int U'_{w'}(u'_{w'})dG'$.

Consider a tax perturbation $\dot{\mathcal{T}}$. The envelope theorem continues to characterize the change in the family surplus, $\dot{v} = -\dot{\mathcal{T}}$. The behavioral responses are now characterized by group-specific behavioral

elasticities, $(\xi, \zeta)$ and $(\xi', \zeta')$, so that (24) reads as

$$\frac{\dot{y}}{y} = -\frac{\xi}{1-\partial\mathcal{T}/\partial y}\frac{\partial\dot{\mathcal{T}}}{\partial y} - \frac{\zeta}{1-\partial\mathcal{T}/\partial y'}\frac{\partial\dot{\mathcal{T}}}{\partial y'}, \quad \frac{\dot{y}'}{y'} = -\frac{\xi'}{1-\partial\mathcal{T}/\partial y'}\frac{\partial\dot{\mathcal{T}}}{\partial y'} - \frac{\zeta'}{1-\partial\mathcal{T}/\partial y}\frac{\partial\dot{\mathcal{T}}}{\partial y}, \tag{65}$$

where $y = y_w(w')$ and $y' = y'_{w'}(w)$ are the earnings of the first- and second-group spouses, and all objects are evaluated at $(y, y')$. These elasticities can be obtained by perturbing the asymmetric spousal first-order conditions, analogously to (59) in the baseline model. As before, the cross-elasticities satisfy Slutsky symmetry,

$$\frac{y\zeta}{1-\partial\mathcal{T}/\partial y'} = \frac{y'\zeta'}{1-\partial\mathcal{T}/\partial y}.$$

Now, perturbing the matching equilibrium defined above by (A)–(C), we can again write

$$\dot{u} = \mathbf{K}\dot{v}, \quad \dot{u}' = \mathbf{K}'\dot{v}, \quad \dot{f}/f = \dot{q}/q = \dot{q}'/q' = \mathbf{M}\dot{v}, \tag{66}$$

for certain linear operators $\mathbf{K}$, $\mathbf{K}'$, $\mathbf{M}$ that can be obtained by differentiating (A)–(C) and $\int U_w(u_w)dG = \int U'_{w'}(u'_{w'})dG'$. This last equation ensures that the perturbed system is uniquely solvable.

Equations (65) and (66) characterize the responses of equilibrium variables to tax perturbations. As in Section 4, one can use them to understand the effects of taxation. For example, the change in revenues can be expressed as

$$\dot{R} = \frac{1}{2}\int \dot{\mathcal{T}}dF + \frac{1}{2}\int\left(\frac{\partial\mathcal{T}}{\partial y}\dot{y} + \frac{\partial\mathcal{T}}{\partial y'}\dot{y}'\right)dF - \frac{1}{2}\int \mathbf{M}^*\mathcal{T}\cdot\dot{\mathcal{T}}dF.$$

One can also study efficient and optimal taxes along the lines of Section 5. For example, assume Pareto weights on individual expected utilities, i.e.,

$$W = \frac{1}{2}\int \alpha U_w(u_w)dG + \frac{1}{2}\int \alpha' U'_{w'}(u'_{w'})dG',$$

where each group's Pareto weights, $\alpha$ and $\alpha'$, are normalized to integrate to one, so that the planner has no preference for or against either group. Then the optimal individual-based, group-specific taxes $T$ and $T'$ are given by

$$\begin{aligned}\frac{\partial T(t)/\partial y}{1-\partial T(t)/\partial y} &= \frac{1}{\bar{\xi}(t)}\frac{1-H(t)}{h(t)t}\mathbb{E}\left[1-\mathbf{K}^*\alpha-\mathbf{K}'^*\alpha'-\mathbf{M}^*\mathcal{T}|y>t\right] \\ &\quad -\mathbb{E}\left[\frac{\zeta}{\bar{\xi}(t)}\frac{\partial T'/\partial y'}{1-\partial T'/\partial y'}|y=t\right],\end{aligned} \tag{67}$$

and similarly for the other group. In contrast, the optimal tax function restricted to be individual-based and identical across the two groups, $T = T'$, is given by

$$\begin{aligned}\frac{\partial T(t)/\partial y}{1-\partial T(t)/\partial y} &= \frac{1}{\bar{\xi}^{\mathrm{p}}(t)}\frac{1-H^{\mathrm{p}}(t)}{h^{\mathrm{p}}(t)t}\mathbb{E}^{\mathrm{p}}\left[1-\mathbf{K}^*\alpha-\mathbf{K}'^*\alpha'-\mathbf{M}^*\mathcal{T}|y>t\right] \\ &\quad -\mathbb{E}^{\mathrm{p}}\left[\frac{\zeta}{\bar{\xi}^{\mathrm{p}}(t)}\frac{\partial T^{\mathfrak{s}}/\partial y}{1-\partial T^{\mathfrak{s}}/\partial y}|y=t\right],\end{aligned} \tag{68}$$

where the superscript p denotes pooling of individuals across the two groups, i.e., $H^{\mathrm{p}} = \frac{1}{2}(H + H')$, $h^{\mathrm{p}} = \frac{1}{2}(h + h')$, and $\bar{\xi}^{\mathrm{p}} = (\bar{\xi}h + \bar{\xi}'h')/(h + h')$. This characterization resembles the one in Proposition 3, with the symmetrized sharing and sorting terms of (36) replaced by averages over individuals of both groups. In particular, if $\alpha = \alpha'$, Pareto weights depend only on productivity, as in (27), and (68) is exactly the formula (36) of Proposition 3 written for the pooled population of individuals of both groups, with the operator $\mathbf{K}$ replaced by its pooled counterpart $\mathbf{K}^{\mathrm{p}}$.

Finally, as for efficiency, the argument of Proposition 2 applies. Group-specific taxes are efficient if and only if there exist Pareto weights $\alpha$ and $\alpha'$, measures over the agents of each group in matches normalized to one, such that both conditions in (67) hold at every earnings level. Unlike in the welfare criterion above, these weights need not put equal total weight on the two groups. The common schedule $T = T'$ is efficient if and only if (68) holds for some such weights.

### 12.1.2 Singles

Consider the baseline economy, but allow agents to remain single and pay taxes $\hat{\mathcal{T}}$. A single individual of type $w$ solves

$$\hat{v}(w) = \max_{l} wl - \hat{\mathcal{T}}(wl) + \hat{\mathcal{N}}(l),$$

where $\hat{\mathcal{N}}$ is the (dis)utility of labor supply of a single individual, and we denote their optimal earnings by $\hat{y}(w)$. The problem of couples is still given by (5), with spousal pecuniary utilities determined in equilibrium. It is convenient to measure those utilities relative to the value of singlehood, so that the utility of type $w$ married to $w'$ is $\hat{v}(w) + u_w(w')$.

In the marriage market, each person's choice set is $W \cup \{\emptyset\}$, where $\emptyset$ denotes singlehood and, after this normalization, has pecuniary payoff zero. We define $U_w$ as in (2) on this enlarged choice set by

$$U_w(u_w) := \mathbb{E}_w \left[ \max \left\{ \max_{w'} u_w(w') + \beta^{-1}\epsilon(w'), 0 \right\} \right],$$

and write $\hat{q}_w = 1 - \int q_w(w')dw'$ for the probability of remaining single.

**Definition.** An equilibrium in the marriage market given the family surplus $v$ and the values of singlehood $\hat{v}$ consists of functions $(u, q)$ such that

(A). $q_w = \partial U_w(u_w)$;

(B). $q_w(w')g(w) = q_{w'}(w)g(w')$;

(C). $u_w(w') + u_{w'}(w) = v(w, w') - \hat{v}(w) - \hat{v}(w')$.

Relative to the baseline, singlehood changes the equilibrium in two ways. First, spouses now split the net surplus of marriage, $v - \hat{v} - \hat{v}^{\mathtt{s}}$, rather than $v$, so taxes on singles affect couples through the value of their outside option. Second, choice probabilities no longer sum to one over partners; the remainder, $\hat{q}_w$, is the probability of remaining single, so the marriage rate is endogenous.

The behavioral responses of singles are characterized by the elasticity of earnings with respect to the retention rate, $\hat{\xi}$, exactly as in Saez (2001): $\dot{\hat{y}}/\hat{y} = -\hat{\xi}\frac{\partial\dot{\hat{\mathcal{T}}}/\partial y}{1-\partial\hat{\mathcal{T}}/\partial y}$. The behavioral responses of couples are the same as in the baseline.

The addition of singles slightly alters perturbations of the matching equilibrium. Specifically, the own matching semi-elasticity now includes substitution toward singlehood,

$$\bar{\gamma}_w(w') = \int \gamma_w(\tilde{w}'|w')d\tilde{w}' + \gamma_w(\emptyset|w').$$

By the envelope theorem, the change in the net surplus of marriage is $-\dot{\mathcal{T}}(y_w(w'), y_w^{\mathfrak{s}}(w')) + \dot{\hat{\mathcal{T}}}(\hat{y}(w)) + \dot{\hat{\mathcal{T}}}(\hat{y}(w'))$. Consequently, the argument of Lemma 1 gives

$$\dot{u} = \mathbf{K}\left(-\dot{\mathcal{T}} + \dot{\hat{\mathcal{T}}} + \dot{\hat{\mathcal{T}}}^{\mathfrak{s}}\right), \qquad \frac{\dot{f}}{f} = \mathbf{M}\left(-\dot{\mathcal{T}} + \dot{\hat{\mathcal{T}}} + \dot{\hat{\mathcal{T}}}^{\mathfrak{s}}\right),$$

where $\mathbf{K}$ and $\mathbf{M}$ are the operators of Lemma 1 built from the amended $\bar{\gamma}$. The marriage rate responds by

$$\dot{\hat{q}}_w g(w) = -\int \dot{f}(w, w')dw'.$$

One can use these expressions to study the effects of taxation, as well as efficient and optimal taxes, as in Sections 4 and 5.

## 12.2 Proof of Lemma 4

Since Conditions (A) and (B) are unchanged, (60) continues to apply. Differentiating (44) gives

$$\frac{\dot{u}_w(w')}{\partial V_w(w')/\partial c} + \frac{\dot{u}_{w'}(w)}{\partial V_{w'}(w)/\partial c} = \dot{v}(w, w').$$

Combining it with (60), we obtain

$$\frac{\dot{u}_w(w')}{\partial V_w(w')/\partial c} = \frac{\bar{\gamma}^c_{w'}(w)\dot{v}(w, w') + \int \gamma_w(\tilde{w}'|w')\dot{u}_w(\tilde{w}')d\tilde{w}' - \int \gamma_{w'}(\tilde{w}|w)\dot{u}_{w'}(\tilde{w})d\tilde{w}}{\bar{\gamma}^c_w(w') + \bar{\gamma}^c_{w'}(w)},$$

and multiplying by $\partial V_w(w')/\partial c$ gives (45). We assume that $\mathbf{I} - \mathbf{A}^c + \mathbf{B}^c$ has a bounded inverse, which holds by Lemma 7 when marginal utility is constant and, by continuity, near that case. Under this assumption, the argument in the proof of Lemma 1 gives the lemma with $\mathbf{K}^c = (\mathbf{I} - \mathbf{A}^c + \mathbf{B}^c)^{-1}\mathbf{\Theta}^c$ and $\mathbf{M}^c = \mathbf{C}\mathbf{K}^c$.

## 12.3 Taxation and sorting in firms

In this section, we illustrate how to solve for the responses $\dot{u}$ and $\dot{y}$ in (51) and (52) in the FC-MNL model described in Section 9. In this model, $\bar{\gamma}$ equals

$$\bar{\gamma}_w(x) = \beta + \frac{\beta(1-\sigma)}{\sigma q_w(x)} \int \pi_w(\tilde{x}|x)p(x, \tilde{x})p(\tilde{x}, x)d\tilde{x},$$

and it depends on the menu of pecuniary utilities unless $\sigma = 1$. The response of $\bar{\gamma}_w$, that is, its (Fréchet) derivative with respect to the pecuniary payoffs evaluated at the equilibrium payoffs, can be shown to be given by

$$\begin{aligned}
\dot{\bar{\gamma}}_w(x) &= \frac{\beta^2(1-\sigma)}{\sigma q_w(x)} \int \pi_w(\tilde{x}|x) p(x,\tilde{x}) p(\tilde{x},x) \left( p(x,\tilde{x}) + \frac{p(\tilde{x},x) - p(x,\tilde{x})}{\sigma} \right) \dot{u}_w(x) d\tilde{x} \\
&+ \frac{\beta^2(1-\sigma)}{\sigma q_w(x)} \int \pi_w(\tilde{x}|x) p(x,\tilde{x}) p(\tilde{x},x) \left( p(\tilde{x},x) + \frac{p(x,\tilde{x}) - p(\tilde{x},x)}{\sigma} \right) \dot{u}_w(\tilde{x}) d\tilde{x} \\
&+ \beta(\beta - \bar{\gamma}_w(x)) \int q_w(\tilde{x}) \dot{u}_w(\tilde{x}) d\tilde{x} + (\beta - \bar{\gamma}_w(x)) \int \gamma_w(\tilde{x}|x) \left( \dot{u}_w(x) - \dot{u}_w(\tilde{x}) \right) d\tilde{x}.
\end{aligned}$$

Then, eliminating $\dot{y}_x(w)$ from (51) and (52) gives

$$\dot{u}_w(x) = \omega_w(x) \dot{\bar{\gamma}}_w(x) - \vartheta_w(x) \dot{T}(y_x(w)) - \iota_w(x) \frac{\partial}{\partial y} \dot{T}(y_x(w)),$$

for certain coefficients $\omega_w(x)$, $\vartheta_w(x)$, and $\iota_w(x)$. Combining these two expressions, we obtain a Fredholm integral equation of the second kind for $\dot{u}$. Once this response has been found, we can find $\dot{\bar{\gamma}}$, $\dot{y}$, and $\dot{q}$ using the expression for $\dot{\bar{\gamma}}$ above, (51), and (49), respectively.